Exercise 5.17

(a) For a whole life annuity-due on (65), we can write the present value random variable of the benefits as

$$Y = \ddot{a}_{\overline{K+1}|},$$

where K is the curtate future lifetime of (65) with probability mass

$$\Pr[K = k] = {}_kp_{65}\, q_{65+k}.$$

The following R code calculates the expected value and variance of Y based on the Standard Ultimate Survival Model with $i = 5\%$:

```
# whole life annuity-due on (65)
A <- .00022
B <- 2.7*10^(-6)
c <- 1.124
surv <- function(x){
exp(-A*x-(B*(c^x-1)/log(c)))}
x <- 65:137
px <- surv(x+1)/surv(x)
qx <- 1-px
int <- .05
v <- 1/(1+int)
vcum <- v^(0:(length(x)-1))
y <- cumsum(vcum)
prob <- cumprod(c(1,px[-length(px)]))*qx
prob[length(prob)] <- 1 - sum(prob[-length(prob)])
EY <- sum(y*prob)
EY2 <- sum(y^2 * prob)
VarY <- EY2 - EY^2
```

This produces the results:

```
> EY
[1] 13.54979
> VarY
[1] 12.49732
```

(b) With a 10-year annuity guarantee, we can write the present value random variable of the benefits as

$$Y = \ddot{a}_{\overline{10}|} I(K \leq 9) + \ddot{a}_{\overline{K+1}|} I(K > 9).$$

The following R code calculates the expected value and variance of Y based on the Standard Ultimate Survival Model with $i = 5\%$:

```
# whole life annuity-due on (65), with 10 year guarantee
# replace the first 10 years of benefit with a 10-year annuity-due, guaranteed
y[1:10] <- y[10]
EY <- sum(y*prob)
EY2 <- sum(y^2 * prob)
VarY <- EY2 - EY^2
```

This produces the results:

```
> EY
[1] 13.81410
> VarY
[1] 8.379656
```

A life annuity with a guarantee leads to some fixed, but more expensive and with less variable benefit payments. Hence, the life annuity with guarantee has a higher expectation but lower variance.