Exercise 5.14

Based on the Standard Ultimate Survival Model with $i = 5\%$, the following values have been calculated:

x	$10000 \cdot A_x$	$10000 \cdot {}^2A_x$	$\ddot{a}_x$
60	2902.82	1083.41	14.904074
70	4281.76	2146.67	12.008303
80	5929.33	3813.41	8.548406

Denote by $Y_i(60)$, $Y_i(70)$ and $Y_i(80)$ the present value random variables of the annuities respectively for ages 60, 70 and 80. Then we have

$$\mathrm{E}[Y_i(60)] = 10000(14.904074) = 149040.7,$$

$$\mathrm{Var}[Y_i(60)] = \frac{10000(1083.41) - (2902.82)^2}{(.05/1.05)^2} = 1061811597,$$

$$\mathrm{E}[Y_i(70)] = 10000(12.008303) = 120083.0,$$

$$\mathrm{Var}[Y_i(70)] = \frac{10000(2146.67) - (4281.76)^2}{(.05/1.05)^2} = 1381755004,$$

$$\mathrm{E}[Y_i(80)] = 10000(8.548406) = 85484.06,$$

and

$$\mathrm{Var}[Y_i(80)] = \frac{10000(3813.41) - (5929.33)^2}{(.05/1.05)^2} = 1312921276.$$

(a) The total outgo on the annuities can be computed based on

$$Y = \sum_{i=1}^{40} Y_i(60) + \sum_{i=1}^{30} Y_i(70) + \sum_{i=1}^{10} Y_i(80).$$

Its expected value is thus

$$\mathrm{E}[Y] = 40(149040.7) + 30(120083.0) + 10(85484.06) = 10418961.$$

(b) Its standard deviation is

$$\mathrm{SD}[Y] = \sqrt{40(1061811597) + 30(1381755004) + 10(1312921276)} = 311535.4.$$

(c) Using the Normal approximation, the 95-th percentile of Y is

$$y_{0.95} = \mathrm{E}[Y] + 1.645 \cdot \mathrm{SD}[Y] = 10418961 + 1.645(311535.4) = 10931437.$$