Exercise 4.7

(a) For an n-year increasing term insurance, the benefit starts at \$1 and increases by \$1 each year thereafter, provided death occurs within the first n years.

$$\begin{aligned}(IA)^{1}_{x:\overline{n|}} &= \sum_{k=0}^{n-1}(k+1)v^{k+1}\,{}_{k|}q_x \\ &= vq_x + \sum_{k=1}^{n-1}(k+1)v^{k+1}\,{}_{k}p_x\,q_{x+k}\end{aligned}$$

Applying change of variable $s = k-1$ so that $k = s+1$, we have

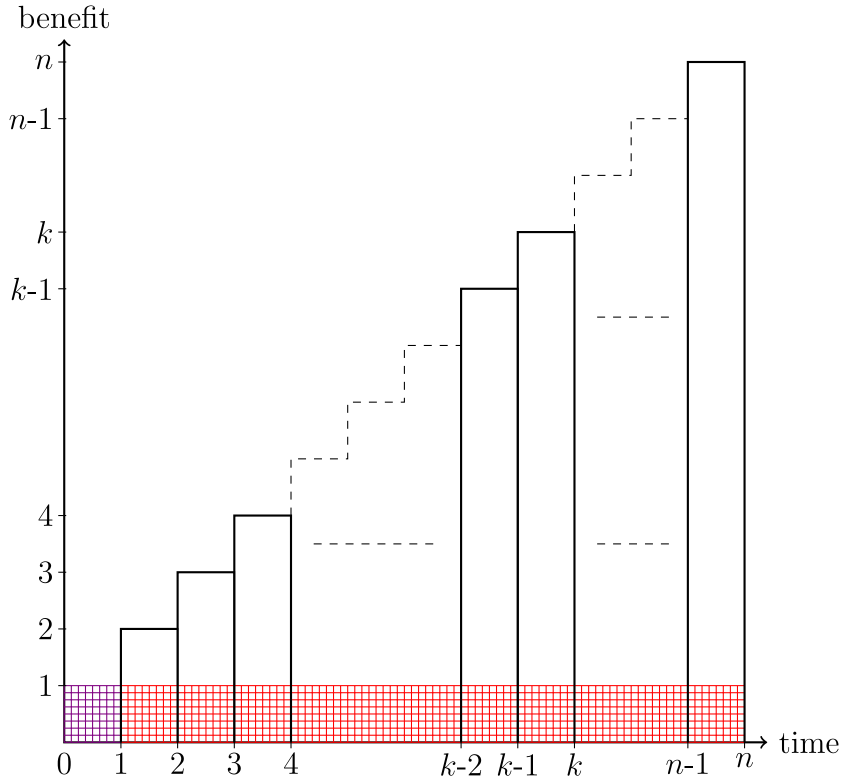
$$\begin{aligned}(IA)^{1}_{x:\overline{n|}} &= vq_x + \sum_{s=0}^{n-2}(s+2)v^{s+2}\,{}_{s+1}p_x\,q_{x+s+1} \\ &= vq_x + \sum_{s=0}^{n-2}(s+2)v^{s+2}\,{}_{s+1}p_x\,q_{x+s+1} \\ &= vq_x + \sum_{s=0}^{n-2}(s+2)v^{s+2}\,p_x\,{}_{s}p_{x+1}\,q_{x+1+s} \\ &= vq_x + vp_x\sum_{s=0}^{n-2}(s+2)v^{s+1}\,{}_{s}p_{x+1}\,q_{x+1+s} \\ &= vq_x + vp_x\left[\sum_{s=0}^{n-2}(s+1)v^{s+1}\,{}_{s}p_{x+1}\,q_{x+1+s} + \sum_{s=0}^{n-2}v^{s+1}\,{}_{s}p_{x+1}\,q_{x+1+s}\right] \\ &= vq_x + vp_x\left[\sum_{s=0}^{n-2}(s+1)v^{s+1}\,{}_{s|}q_{x+1} + \sum_{s=0}^{n-2}v^{s+1}\,{}_{s|}q_{x+1}\right] \\ &= vq_x + vp_x\left[(IA)^{1}_{x+1:\overline{n-1|}} + A^{1}_{x+1:\overline{n-1|}}\right],\end{aligned}$$

which shows the desired result.

(b) An n-year increasing term insurance is equivalent to a payment of \$1 if death occurs in the first year. This is the first term, vq_x, from the equation above. If the policyholder survives the first year and therefore reaches age $x+1$, the benefit consists of

- an $n-1$ year term insurance of \$1 payable to $(x+1)$: $A^{1}_{x+1:\overline{n-1|}}$, and
- an $n-1$ year increasing term insurance of \$1 payable to $(x+1)$: $(IA)^{1}_{x+1:\overline{n-1|}}$.

These benefits have to be discounted with life by multiplying vp_x. This interpretation is best visualized with the figure below. This picture shows the benefit (the purple grid) if death occurs in the first year, and if death does not occur in the first year, the benefit consists of an $n-1$ year term insurance of \$1 beginning at age $x+1$ (the red grid) and an $n-1$ year increasing term insurance beginning at age $x+1$ (the unshaded vertical bars).



(c) Part (a) can be extended to an increasing whole life insurance with

$$(IA)_x = vq_x + vp_x\left[(IA)_{x+1} + A_{x+1}\right].$$

Thus for a life (50), we have

$$(IA)_{51} = \frac{(IA)_{50} - vq_{50}}{vp_{50}} - A_{51}.$$

By noting that $A^{\ 1}_{50:\overline{1|}} = vq_{50}$, then $vp_{50} = v(1-q_{50}) = v - A^{\ 1}_{50:\overline{1|}}$. This leads us to

$$\begin{aligned}(IA)_{51} &= \frac{(IA)_{50} - A^{\ 1}_{50:\overline{1|}}}{v - A^{\ 1}_{50:\overline{1|}}} - A_{51} \\ &= \frac{4.99675 - 0.00558}{(1/1.06) - 0.00558} - 0.24905 \\ &= 5.073069\end{aligned}$$