Exercise 4.4

Although not clearly stated in the problem, we assume that benefits are payable at the end of the year of death. With the reversionary bonus, we find that the benefit payment is

$$b_{K+1} = 100000(1.03)^K, \quad \text{for } K = 0, 1, 2, \ldots$$

and the discount function is

$$v_{K+1} = \frac{1}{(1.05)^{K+1}}, \quad \text{for } K = 0, 1, 2, \ldots$$

where K refers to the curtate future lifetime of (30). This leads us to the Actuarial Present Value of the benefits:

$$\begin{aligned} \text{APV(benefits)} &= \text{E}\left[b_{K+1}v_{K+1}\right] = 100000 \sum_{k=0}^{\infty} \frac{(1.03)^k}{(1.05)^{k+1}} {}_{k|}q_{30} \\ &= \frac{100000}{1.03} \sum_{k=0}^{\infty} \frac{1}{(1.05/1.03)^{k+1}} {}_{k|}q_{30} = \frac{100000}{1.03} \left(A_{30}\right)_{i^*} \end{aligned}$$

where $(A_{30})_{i^*}$ is the APV of a whole life insurance of \$1 payable at the end of the year of death of (30) evaluated at the interest rate

$$i^* = (1.05/1.03) - 1 = 0.01941748.$$

To evaluate this APV based on the Standard Ultimate Survival Model, the following R code has been written

```
A <- 0.00022
B <- 2.7*10^(-6)
c <- 1.124
surv <- function(x){
exp(-A*x-(B*(c^x-1)/log(c)))}
x <- 30:118
px <- surv(x+1)/surv(x)
qx <- 1-px
int <- (1.05/1.03)-1
v <- 1/(1+int)
vcum <- v^(1:length(x))
A30s <- 100000*(1/1.03)*sum(vcum*cumprod(c(1,px[-length(px)]))*qx)
A30s
```

to produce the following result:

```
> A30s
[1] 33569.47
```