Exercise 4.16

(a)

$$\begin{aligned} A_{[40]+1:\overline{4}|} &= \sum_{k=0}^{3} v^{k+1}{}_{k|}q_{[40]+1} + v^4{}_4p_{[40]+1} = \sum_{k=0}^{3} v^{k+1}\frac{d_{[40]+1+k}}{\ell_{[40]+1}} + v^4\frac{\ell_{45}}{\ell_{[40]+1}} \\ &= \frac{1}{\ell_{[40]+1}}\left(vd_{[40]+1} + v^2d_{[40]+2} + v^3d_{[40]+3} + v^4\ell_{44}\right) \\ &= \frac{1}{99899}\left[\frac{1}{1.06}(99899-99724) + \frac{1}{1.06^2}(99724-99520)\right. \\ &\qquad \left. + \frac{1}{1.06^3}(99520-99288) + \frac{1}{1.06^4}99288\right] \\ &= 0.792669 \end{aligned}$$

(b) Denote by Z the required present value random variable. The following table shows the details of the calculations:

k	$\Pr[K_{[40]}=k]$	z	$z\Pr[K_{[40]}=k]$	z^2	$z^2\Pr[K_{[40]}=k]$
0	0.00101	0.00	0.0000	0	0
1	0.00175	88999.64	155.7494	7920936632	13861639
2	0.00204	83961.93	171.2823	7049605404	14381195
3	0.00232	79209.37	183.7657	6274123713	14555967
4	0.00255	74725.82	190.5508	5583947769	14239067
≥ 5	0.99033	0.00	0.0000	0	0
sum	1.00000		701.3483		57037868

The values in the table can be verified by noting that $\Pr[K_{[40]}=k] = \dfrac{d_{[40]+k}}{\ell_{[40]}}$ and that the values of Z are 0 in the first year (because it is one year deferred), $100000v^2$ in the second year, and so forth, with no payment if [40] survives to reach 5 years.

Thus, we find from this table that

$$\mathrm{E}[Z] = \sum_{k=0}^{5} z\Pr[K_{[40]}=k] = 701.3483 \quad\text{and}\quad \mathrm{E}[Z^2] = \sum_{k=0}^{5} z^2\Pr[K_{[40]}=k] = 57037868$$

so that

$$\mathrm{Var}[Z] = \mathrm{E}[Z^2] - (\mathrm{E}[Z])^2 = 57037868 - (701.3483)^2 = 56545979.$$

Finally, we have the standard deviation: $\mathrm{SD}[Z] = \sqrt{\mathrm{Var}[Z]} = \sqrt{56545979} = 7519.706.$

(c) Looking up from the table in (b), we find that

$$\Pr[Z \leq 85000] = 1 - \Pr[Z > 85000] = 1 - \Pr\left[K_{[40]} = 1\right] = 1 - 0.00175 = 0.99825.$$