Exercise 4.13

We can write X as $X = v^{\min(T,n)} = v^T I(T \leq n) + v^n I(T > n)$ and Y as $Y = v^T I(T \leq n)$. Now denote the present random variable associated with an n-year pure endowment of a unit issued to (x) by W so that $W = v^n I(T > n)$. We therefore have that

$$\mathrm{E}[W] = v^n {}_n p_x = 0.30(0.80) = 0.24$$

and

$$\mathrm{Var}[W] = v^{2n} {}_n p_x (1 - {}_n p_x) = (0.30)^2 (0.80)(0.20) = 0.0144.$$

Clearly, $X = Y + W$ so that

$$\mathrm{Var}[X] = \mathrm{Var}[Y] + \mathrm{Var}[W] + 2\mathrm{Cov}[Y, W],$$

where $\mathrm{Cov}[Y, W] = \mathrm{E}[YW] - \mathrm{E}[Y]\mathrm{E}[W] = 0 - 0.04(0.24) = -0.0096$. Solving for the variance of Y, we have

$$\mathrm{Var}[Y] = \mathrm{Var}[X] - \mathrm{Var}[W] - 2\mathrm{Cov}[Y, W] = 0.0052 - 0.0144 + 2(0.0096) = 0.01.$$