Exercise 3.6

First we note that ${}_{2|3}q_{[50]+1} = {}_{2}p_{[50]+1} \cdot {}_{3}q_{53}$ from which we derive

$$ {}_{3}p_{53} = 1 - {}_{3}q_{53} = 1 - \frac{{}_{2|3}q_{[50]+1}}{{}_{2}p_{[50]+1}}. $$

It is easy to verify the following holds:

$$ {}_{3}p_{50} = p_{[50]} \cdot {}_{2}p_{[50]+1} = {}_{2}p_{[50]} \cdot p_{[50]+2}, $$

from which we see that

$$ {}_{2}p_{[50]+1} = \frac{{}_{2}p_{[50]} \cdot p_{[50]+2}}{p_{[50]}}. $$

Because

$$ {}_{2|}q_{[50]} = {}_{2}p_{[50]} \cdot q_{[50]+2}, $$

then it follows that

$$ p_{[50]+2} = 1 - \frac{{}_{2|}q_{[50]}}{{}_{2}p_{[50]}} = \frac{{}_{2}p_{[50]} - {}_{2|}q_{[50]}}{{}_{2}p_{[50]}}, $$

and that

$$ {}_{2}p_{[50]+1} = \frac{{}_{2}p_{[50]} - {}_{2|}q_{[50]}}{p_{[50]}} = \frac{0.96411 - 0.02410}{1 - 0.01601} = 0.9553044. $$

Finally, we have

$$ {}_{3}p_{53} = 1 - \frac{0.09272}{0.9553044} = 0.902942. $$