Exercise 2.12

(a) For Makeham's law, it can easily be verified that

$$p_x = \exp\left\{-\left[A + \frac{B}{\log(c)}c^x(c-1)\right]\right\}.$$

The following R code produces a table of p_x for $x = 0$ to $x = 130$:

```
A <- .0001
B <- .00035
c <- 1.075

px <- function (x) {
temp <- A + (B/log(c))*c^x*(c-1)
exp(-temp)}
x <- 0:130
p <- px(x)
output <- cbind(x,round(p,5))
colnames(output) <- c("x","px")
print(output)
```

This gives the output

```
> print(output)
        x      px
  [1,]   0 0.99954
  [2,]   1 0.99951
  [3,]   2 0.99948
  [4,]   3 0.99945
  [5,]   4 0.99942
  [6,]   5 0.99938
  [7,]   6 0.99934
  [8,]   7 0.99930
  [9,]   8 0.99925
 [10,]   9 0.99920
 [11,]  10 0.99915
 [12,]  11 0.99910
 [13,]  12 0.99904
 [14,]  13 0.99897
 [15,]  14 0.99890
 [16,]  15 0.99883
 [17,]  16 0.99875
 [18,]  17 0.99866
 [19,]  18 0.99857
 [20,]  19 0.99847
 [21,]  20 0.99836
```

```
[22,]  21 0.99824
[23,]  22 0.99812
[24,]  23 0.99799
[25,]  24 0.99784
[26,]  25 0.99769
[27,]  26 0.99752
[28,]  27 0.99735
[29,]  28 0.99715
[30,]  29 0.99695
[31,]  30 0.99673
[32,]  31 0.99649
[33,]  32 0.99623
[34,]  33 0.99596
[35,]  34 0.99567
[36,]  35 0.99535
[37,]  36 0.99501
[38,]  37 0.99464
[39,]  38 0.99425
[40,]  39 0.99383
[41,]  40 0.99337
[42,]  41 0.99288
[43,]  42 0.99236
[44,]  43 0.99180
[45,]  44 0.99119
[46,]  45 0.99054
[47,]  46 0.98984
[48,]  47 0.98909
[49,]  48 0.98829
[50,]  49 0.98742
[51,]  50 0.98649
[52,]  51 0.98550
[53,]  52 0.98442
[54,]  53 0.98327
[55,]  54 0.98204
[56,]  55 0.98071
[57,]  56 0.97929
[58,]  57 0.97776
[59,]  58 0.97612
[60,]  59 0.97435
[61,]  60 0.97247
[62,]  61 0.97044
[63,]  62 0.96826
[64,]  63 0.96593
[65,]  64 0.96343
[66,]  65 0.96075
[67,]  66 0.95788
```

```
 [68,]  67 0.95480
 [69,]  68 0.95150
 [70,]  69 0.94796
 [71,]  70 0.94418
 [72,]  71 0.94013
 [73,]  72 0.93579
 [74,]  73 0.93115
 [75,]  74 0.92619
 [76,]  75 0.92089
 [77,]  76 0.91522
 [78,]  77 0.90916
 [79,]  78 0.90270
 [80,]  79 0.89580
 [81,]  80 0.88845
 [82,]  81 0.88061
 [83,]  82 0.87226
 [84,]  83 0.86337
 [85,]  84 0.85391
 [86,]  85 0.84387
 [87,]  86 0.83320
 [88,]  87 0.82188
 [89,]  88 0.80988
 [90,]  89 0.79718
 [91,]  90 0.78374
 [92,]  91 0.76956
 [93,]  92 0.75459
 [94,]  93 0.73883
 [95,]  94 0.72225
 [96,]  95 0.70484
 [97,]  96 0.68660
 [98,]  97 0.66751
 [99,]  98 0.64758
[100,]  99 0.62683
[101,] 100 0.60525
[102,] 101 0.58289
[103,] 102 0.55977
[104,] 103 0.53593
[105,] 104 0.51144
[106,] 105 0.48636
[107,] 106 0.46077
[108,] 107 0.43476
[109,] 108 0.40843
[110,] 109 0.38191
[111,] 110 0.35531
[112,] 111 0.32878
[113,] 112 0.30247
```

```
[114,] 113 0.27652
[115,] 114 0.25111
[116,] 115 0.22639
[117,] 116 0.20252
[118,] 117 0.17967
[119,] 118 0.15796
[120,] 119 0.13755
[121,] 120 0.11853
[122,] 121 0.10101
[123,] 122 0.08506
[124,] 123 0.07070
[125,] 124 0.05796
[126,] 125 0.04682
[127,] 126 0.03721
[128,] 127 0.02907
[129,] 128 0.02230
[130,] 129 0.01676
[131,] 130 0.01234
```

(b) To find the age last birthday at which (70) is most likely to die, we need to evaluate the deferred probability

$$ {}_{t|}q_{70} = {}_{t}p_{70} - {}_{t+1}p_{70}. $$

The R code to generate these probabilities for different values of t is given by

```
tp70 <- function (t) {
temp <- A*t + (B/log(c))*c^70*(c^t-1)
exp(-temp)}
tbarq70 <- function (t) {
temp <- tp70(t) - tp70(t+1)
temp}
x <- 70:130
t <- rev(130-x)
q <- tbarq70(t)
output <- cbind(x,t,round(q,5))
colnames(output) <- c("x","t","t|q70")
print(output)
```

This gives the output

```
> print(output)
        x  t    t|q70
 [1,]  70  0 0.05582
 [2,]  71  1 0.05653
 [3,]  72  2 0.05700
 [4,]  73  3 0.05719
 [5,]  74  4 0.05709
```

```
 [6,]  75  5 0.05668
 [7,]  76  6 0.05593
 [8,]  77  7 0.05484
 [9,]  78  8 0.05341
[10,]  79  9 0.05163
[11,]  80 10 0.04952
[12,]  81 11 0.04708
[13,]  82 12 0.04436
[14,]  83 13 0.04139
[15,]  84 14 0.03821
[16,]  85 15 0.03487
[17,]  86 16 0.03144
[18,]  87 17 0.02797
[19,]  88 18 0.02454
[20,]  89 19 0.02120
[21,]  90 20 0.01802
[22,]  91 21 0.01505
[23,]  92 22 0.01233
[24,]  93 23 0.00990
[25,]  94 24 0.00778
[26,]  95 25 0.00597
[27,]  96 26 0.00447
[28,]  97 27 0.00326
[29,]  98 28 0.00230
[30,]  99 29 0.00158
[31,] 100 30 0.00105
[32,] 101 31 0.00067
[33,] 102 32 0.00041
[34,] 103 33 0.00024
[35,] 104 34 0.00014
[36,] 105 35 0.00007
[37,] 106 36 0.00004
[38,] 107 37 0.00002
[39,] 108 38 0.00001
[40,] 109 39 0.00000
[41,] 110 40 0.00000
[42,] 111 41 0.00000
[43,] 112 42 0.00000
[44,] 113 43 0.00000
[45,] 114 44 0.00000
[46,] 115 45 0.00000
[47,] 116 46 0.00000
[48,] 117 47 0.00000
[49,] 118 48 0.00000
[50,] 119 49 0.00000
[51,] 120 50 0.00000
```

```
[52,] 121 51 0.00000
[53,] 122 52 0.00000
[54,] 123 53 0.00000
[55,] 124 54 0.00000
[56,] 125 55 0.00000
[57,] 126 56 0.00000
[58,] 127 57 0.00000
[59,] 128 58 0.00000
[60,] 129 59 0.00000
[61,] 130 60 0.00000
```

According to this output, the age last birthday with the highest deferred probability of death is 73.

(c) It can be shown that

$$ {}_kp_{70} = \exp\left\{ - \left[Ak + \frac{B}{\log(c)} c^{70}(c^k - 1) \right] \right\}. $$

The following R code produces a table of ${}_kp_{70}$ for $k = 1$ to $k = 70$:

```
tp70 <- function (t) {
temp <- A*t + (B/log(c))*c^70*(c^t-1)
exp(-temp)}
x <- 70:130
t <- rev(130-x)
p70 <- tp70(t)[-1]
e70 <- sum(p70)
e70
```

The output:

```
> e70
[1] 9.338684
```

(d) To evaluate the integral for the complete expectation of life for (70), we use again repeated application of the Simpson's rule following the same logic as was done in Exercise 2.4 (d). We have the R code:

```
exc <- function(tol) {
a<-0
h<-.25
k<-0
v1 <- (h/3)*(tp70(a) + 4* tp70(a+h) + tp70(a+2*h))
v <- v1
while (v1 > tol) {
k <- k+2
lim1 <- a+k*h
```

```
mid <- a+(k+1)*h
lim2 <- a+(k+2)*h
v1 <- (h/3)*(tp70(lim1) + 4*tp70(mid) + tp70(lim2))
v <- v + v1}
v}
tol <- 10^(-50)
ec70 <- exc(tol)
ec70
```

The output:

```
> ec70
[1] 9.834068
```