Exercise 2.1

(a) The probability that a newborn life dies before age 60 is given by

$$\Pr[T_0 \le 60] = F_0(60) = 1 - (1 - 60/105)^{1/5} = 1 - (45/105)^{1/5} = 1 - (3/7)^{1/5} = 0.1558791.$$

(b) The probability that (30) survives to at least age 70 is

$$\Pr[T_{30} > 40] = \frac{\Pr[T_0 > 70]}{\Pr[T_0 > 30]} = \frac{1 - F_0(70)}{1 - F_0(30)} = \frac{35^{1/5}}{75^{1/5}} = \left(\frac{7}{15}\right)^{1/5} = 0.8586207.$$

(c) The probability that (20) dies between 90 and 100 is

$$\Pr[70 < T_{20} \le 80] = \frac{\Pr[90 < T_0 \le 100]}{\Pr[T_0 > 20]} = \frac{F_0(100) - F_0(90)}{1 - F_0(20)} = \frac{15^{1/5} - 5^{1/5}}{85^{1/5}} = 0.1394344.$$

(d) First, derive the form of the force of mortality:

$$\mu_x = \frac{f_0(x)}{1 - F_0(x)} = \frac{dF_0(x)/dx}{1 - F_0(x)} = \frac{\frac{1}{5}\left(1 - \frac{x}{105}\right)^{-4/5}\left(\frac{1}{105}\right)}{\left(1 - \frac{x}{105}\right)^{1/5}} = \frac{1}{5(105 - x)}.$$

Thus, $\mu_{50} = \dfrac{1}{5(55)} = 0.003636364.$

(e) The median future lifetime of (50) is the solution m to

$$\Pr[T_{50} > m] = \frac{1}{2} = \left(1 - \frac{m}{55}\right)^{1/5}.$$

This leads us to $m = 55[1 - (1/2)^5] = 53.28125.$

(f) For a person currently age 50, his survival function is

$$ {}_tp_{50} = \Pr[T_{50} > t] = \frac{\Pr[T_0 > 50 + t]}{\Pr[T_0 > 50]} = \left(\frac{55 - t}{55}\right)^{1/5} = \left(1 - \frac{t}{55}\right)^{1/5},$$

for $0 \le t \le 55$. His complete expectation of life is therefore

$$\mathring{e}_{50} = \int_0^{55} {}_tp_{50}\,dt = \int_0^{55} \left(1 - \frac{t}{55}\right)^{1/5} dt = 55 \int_0^1 u^{1/5} du = 55(5/6) = 45.83333.$$

(g) The curtate expectation of life at age 50 is

$$e_{50} = \sum_{k=1}^{55} {}_kp_{50} = \sum_{k=1}^{55} \left(1 - \frac{k}{55}\right)^{1/5} = \left(\frac{54}{55}\right)^{1/5} + \left(\frac{53}{55}\right)^{1/5} + \cdots + \left(\frac{1}{55}\right)^{1/5} = 45.17675.$$

The sum above can be done in an R program as follows:

```
> k <- 1:54
> e <- (k/55)^(1/5)
> sum(e)
[1] 45.17675
```