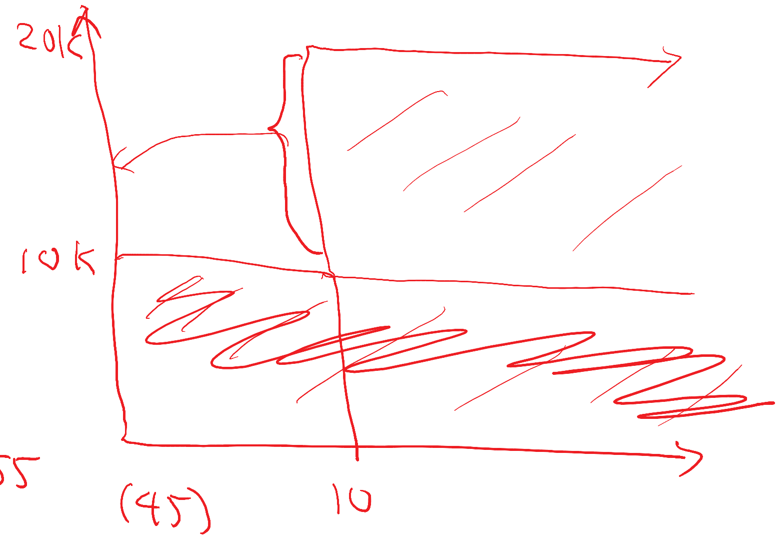
<u>Q1</u>

APV(insurance)

$$= 10{,}000 \, \underbrace{A_{45}}_{.20120} + 10{,}000 \, \underbrace{{}_{10}E_{45}}_{.52652} \, \underbrace{A_{55}}_{.30514}$$

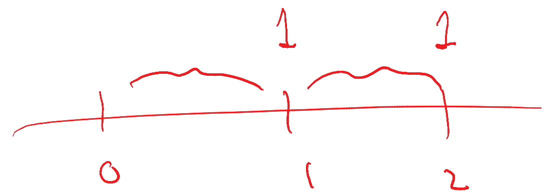


$$= \underline{\underline{3618.623}}$$

Q2   A   E

$E[Z] = E[Z|A]\Pr(A) + E[Z|B]\Pr(B)$

$A \cup B$



$${}^{ns}A^{1}_{x:\overline{2}|} = v\,{}^{ns}q_x + v^2\,{}^{ns}p_x\,{}^{ns}q_{x+1}$$

$$= \frac{1}{1.03}(.02) + \frac{1}{1.03^2}(.98)(.04)$$

$$= .05636724$$

$${}^{sm}A^{1}_{x:\overline{2}|} = v\,\underbrace{{}^{sm}q_x}_{.08} + v^2\,\underbrace{{}^{sm}p_x}_{.92}\,\underbrace{{}^{sm}q_{x+1}}_{.15} = .2077481$$

$$A^{1}_{x:\overline{2}|} = .05636724(.80) + .2077481(.20) = .08664342$$

$APV(\text{benefits})$

i: 2% (65 to 66), 3% (66 to 67), 6% (from 67)

q: .03 (65 to 66), ILT (from 66)

$= 100\, A_{65}$

$$= 100\Big[\underbrace{v}_{\frac{1}{1.02}}\ \underbrace{q_{65}}_{.03} + \underbrace{v^2}_{\frac{1}{1.02}\frac{1}{1.03}}\ \underbrace{p_{65}}_{.97}\ \underbrace{q_{66}^{ILT}}_{\frac{23.29}{1000}} + \underbrace{v^2\, p_{65}}_{{}_2E_{65}}\ \underbrace{p_{66}^{ILT}}_{\left(1-\frac{23.29}{1000}\right)}\ \underbrace{A_{67}^{ILT}}_{.46947}\Big]$$

$= \underline{\underline{47.4273}}$

Q4

$\ddot{a}_{60:\overline{10}|} = 6.5$

$A^{\ \ 1}_{60:\overline{10}|} = .08$

$v = .92$

$$A_{60:\overline{10}|} = 1 - d\,\ddot{a}_{60:\overline{10}|}$$

$(1-v)$

$$= 1-(1-.92)(6.5) = 0.48$$

$$\underbrace{A^{\ \ 1}_{60:\overline{10}|}}_{.08} + \underbrace{A_{60:\overline{10}|}^{\ \ \ \ \ 1}}_{0.40}$$

$${}_{10}E_{60} = A_{60:\overline{10}|}^{\ \ \ \ \ 1}$$

$v @ 2\delta = v^2$

$v = e^{-\delta}$

$v^2 = e^{-2\delta}$

$A_{x:\overline{n}|}^{\ \ \ \ 1} = v^n\,{}_np_x$

$${}^2A_{x:\overline{n}|}^{\ \ \ \ 1} = v^{2n}\,{}_np_x = v^n \cdot \underbrace{v^n\,{}_np_x}_{A_{x:\overline{n}|}^{\ \ \ \ 1}}$$

$$Var[Z] = \underbrace{{}^2A_{60:\overline{10}|}^{\ \ \ \ \ 1}}_{v^{10}\cdot A_{60:\overline{10}|}^{\ \ \ \ \ 1}} - \left(A_{60:\overline{10}|}^{\ \ \ \ \ 1}\right)^2$$

$$= (.92)^{10}(.40) - (.40)^2$$

$$= \cancel{47.42273} \quad 0.0137553\,8$$

$\bar{A}_x = \frac{\mu}{\mu+\delta}$

${}^2\bar{A}_x = \frac{\mu}{\mu+2\delta}$

Q5

UDD $\quad q_x = .05$

$q_{x+1} = .08$

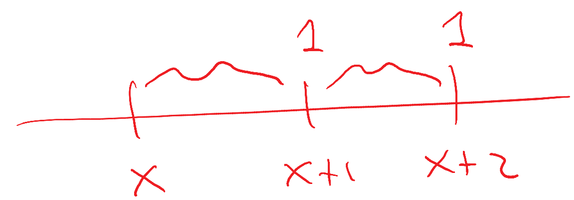
$i = 5\%$

$v = e^{-\delta}$

$v^2 = e^{-2\delta}$

$\bar{A} = \frac{i}{\delta} A$	whole life term deferred
$A^{(m)} = \frac{i}{i^{(m)}} A$	UDD

(a) $\bar{A}^{1}_{x:\overline{2}|} = \frac{i}{\delta} A^{1}_{x:\overline{2}|}$



$e^{\delta} = 1+i$

$\delta = \log(1+i)$

$$= \frac{i}{\delta}\left[v q_x + v^2 p_x q_{x+1} \right]$$

$$= \frac{.05}{\log(1.05)}\left[\frac{1}{1.05}(.05) + \frac{1}{1.05^2}(.95)(.08) \right] = \underline{\underline{.119439}}$$

i @ 2δ

$1+i^* = e^{2\delta}$

$i^* = e^{2\delta} - 1$

$i^* = (1.05)^2 - 1$

$= .1025$

(b) $^{2}\bar{A}^{1}_{x:\overline{2}|} = \frac{\overset{.1025}{i^*}}{2\log(1.05)}\left[\frac{1}{1.05^2}(.05) + \frac{1}{1.05^4}(.95)(.08) \right] = \underline{\underline{.0002697457}}$

(c) $\bar{A}^{1}_{x:\overline{2}|}$ = APV of a 2 year term insurance of 1 payable at the moment of death of (x).

Q6 de Moivre's law $\underline{\underline{\omega}}$ ✓ $Z = PV$ whole life

(50) $T_{50} \sim \text{Uniform}(0, \omega - 50)$ $\downarrow T$

$$\Pr[\underbrace{Z > .0734}] = .95$$

$$\underset{\downarrow}{V^T} = e^{-\delta T} = e^{-.05T}$$

$Z > .0734$

$\Rightarrow e^{-.05T} > .0734$

$\Rightarrow \dfrac{-.05T}{-.05} \cancel{>} < \dfrac{\log(.0734)}{-.05}$

$T < \log(.0734)/-.05$

$$\underbrace{\Pr\left[T < \frac{\log(.0734)}{-.05}\right]}_{\frac{\log(.0734)/-.05}{\omega - 50}} = .95$$

$T \sim (0, \omega - 50)$

$$\Pr[T < a] = \frac{a}{\omega - 50}$$

$$\frac{\log(.0734)/-.05}{\omega - 50} = .95 \Rightarrow \omega = 104.9859 \approx 105 \checkmark$$

Q7 $\ddot{a}_{[57]:\overline{3}|}$

CPT: $\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k \, {}_kp_{x}$

$\ddot{a}_{[x]:\overline{n}|}$ ${}_kp_{[x]}$

$$= 1 + v\,p_{[57]} + v^2\,p_{[57]}\,p_{[57]+1}$$

$$= 1 + v\,\frac{\overset{863}{l_{[57]+1}}}{\underset{868}{l_{[57]}}} + v^2\,\frac{\overset{856}{l_{[57]+2}}}{\underset{868}{l_{[57]}}}$$

$v = \frac{1}{1.05}$, $v^2 = \frac{1}{1.05^2}$

$$= \underline{\underline{2.841385}}$$

Q8 ILT $i = 6\%$

whole life annuity-due

to (50)

benefit $= 100$

$$Var[\ddot{a}_{\overline{K+1}|}]$$

$$\frac{1 - v^{K+1}}{d}$$

$$\frac{1}{d^2}\left[{}^2A_x - (A_x)^2\right]$$

$$Var[Y] = \frac{100^2}{(d)^2}\left[{}^2A_{50} - (A_{50})^2\right]$$

${}^2A_{50} = .09476$ $\qquad A_{50} = .24905$

$$b^2\, Var[\ddot{a}_{\overline{K+1}|}]$$

$d = 1 - v$

$= \frac{i}{1+i}$

$$= \frac{(100)^2}{\left(\frac{.06}{1.06}\right)^2}\left[.09476 - (.24905)^2\right] = \underline{\underline{102,166.80}}$$

Q 9 (Exercise 5.7)

$\ddot{a}_x$

11.2

$_{15|}\ddot{a}_x = 4.5$

$A^{\,1}_{x:\overline{15}|} = 0.212$

$_{15}E_x = 0.255$

Calculate i.

$$\boxed{A_x = 1 - d\,\ddot{a}_x}$$

$$A_{x:\overline{n}|} = 1 - d\,\ddot{a}_{x:\overline{n}|}$$

$$\underbrace{A_{x:\overline{15}|}}_{A^{\,1}_{x:\overline{15}|} + {}_{15}E_x} = 1 - d\,\underbrace{\ddot{a}_{x:\overline{15}|}}_{(\ddot{a}_x - {}_{15|}\ddot{a}_x)}$$

$$(.212 + .255) = 1 - d\underbrace{(11.2 - 4.5)}_{6.7}$$

$$d = \frac{1-(.212+.255)}{6.7}$$

$$= .07955224$$

$$\frac{i}{1+i} = \text{.07955224}$$

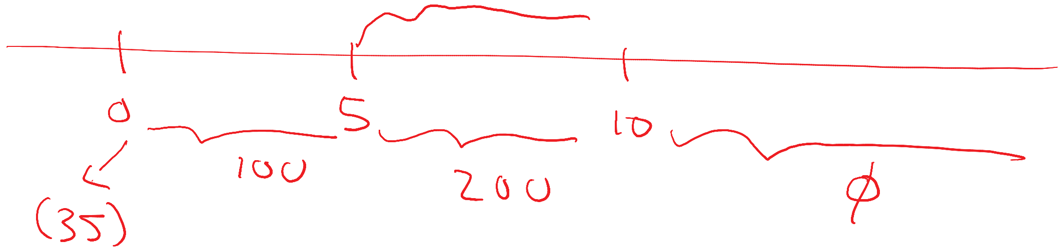
$$\Rightarrow i = \frac{.07955224}{1-.07955224}$$

$$= \underline{\underline{.0864276}}$$

Q10

ILT

$i = 6\%$



$Z =$ PV at age 35

(a) $$Z = \begin{cases} 100 v^{K+1}, & K = 0,1,2,3,4 \quad K<5 \\ 200 v^{K+1}, & K = 5,6,7,8,9 \quad 5 \le K < 10 \\ \phi, & K \geq 10 \end{cases}$$

(b) $$\Pr[Z = \phi] = \Pr[K \geq 10] = {}_{10}p_{35} = \frac{l_{45}}{l_{35}} = \frac{9164051}{9420657} = .9727613$$

(c) $$\underbrace{\Pr[Z > 85]}_{} = {}_2q_{35} = 1 - {}_2p_{35} = 1 - \frac{l_{37}}{l_{35}}$$

$= .00414948$

$100v = 100/1.06 = 94.33962$

$100v^2 = 100/1.06^2 = 88.99964$

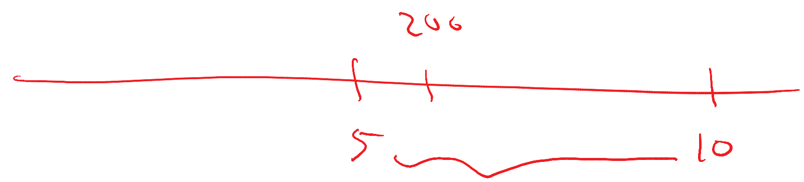
$100v^3 = 100/1.06^3 = 83.96193$

.01997801

$\Pr[Z > 85]$

$= {}_2q_{35} + {}_5p_{35} \cdot {}_5q_{40}$

$= \dfrac{l_{40}}{l_{35}}\left(1 - \dfrac{l_{45}}{l_{40}}\right)$



$200v^6 > 85$

$200v^7$

$200v^8$

$\vdots$

$200v^{10} = 125+$

5MC
Essay
4/5

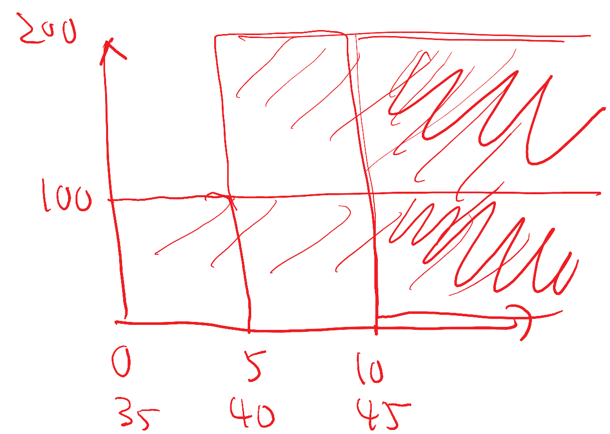
(d) $E[Z] = 100A_{35} + 100\,{}_5E_{35}A_{40}$

.12872 .73873 .16132

$-\ 200\,{}_{10}E_{35}A_{45}$

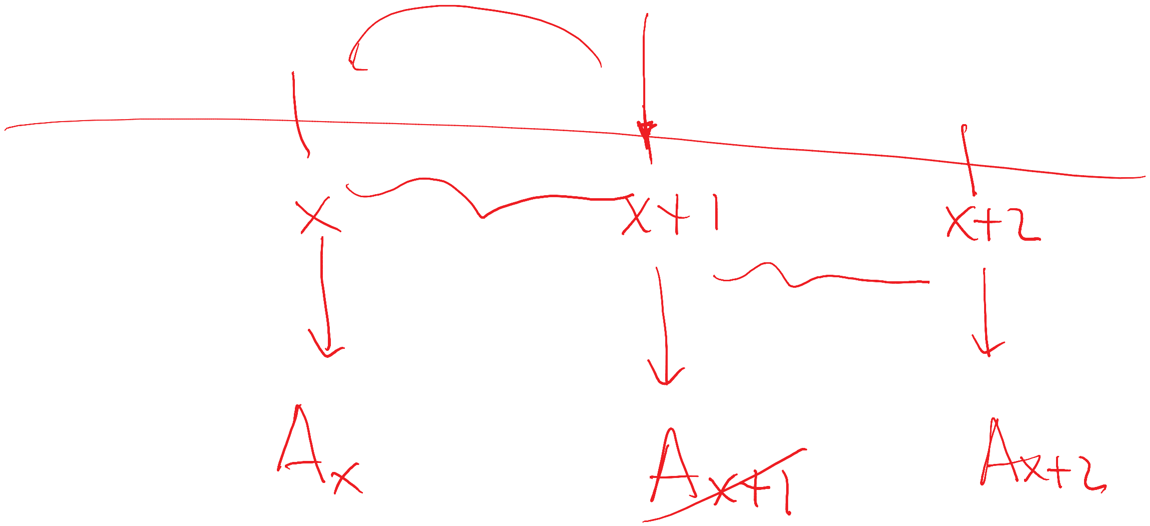
.54318 .20120

$= 2.931629$



try!

(e) $Var[Z] = ?$



$$A_x = v q_x + \underbrace{v p_x}_{{}_1E_x} A_{x+1}$$

$$A_x = v q_x + v^2 p_x q_{x+1} + {}_2E_x A_{x+2}$$