MATH 3630

Actuarial Mathematics I

Sample Test 2

Time Allowed: 1 hour

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: SUGGESTED SOLUTIONS

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

is the APV of a 3-year endowment insurance on (62) with benefit of \$1 payable at the end of year of death

Suppose $i = 4\%$ and mortality follows the following select-and-ultimate table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	x+3
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

$v^3 p_{[62]} p_{[62]+1}$

Calculate $A_{[62]:\overline{3}|}$ and interpret this value.

$$A_{[62]:\overline{3}|} = A^{1}_{[62]:\overline{3}|} + {}_3E_{[62]}$$

$$= v\, q_{[62]} + v^2 p_{[62]}\, q_{[62]+1} + v^3 p_{[62]} p_{[62]+1}\, q_{[62]+2} + v^3 p_{[62]} p_{[62]+1} p_{[62]+2}$$

$$= \underbrace{(1.04)^{-1} 0.11 + (1.04)^{-2} 0.89(0.13) + (1.04)^{-3} 0.89(0.87)}_{0.9010903}$$

alternatively, can also use $\ddot{a}_{62:\overline{3}|} = 1 + v p_{[62]} + v^2 p_{[62]} p_{[62]+1}$

$$= 1 + (1.04)^{-1} 0.89 + (1.04)^{-2} 0.89(0.87)$$

$$= 2.571653$$

Then use relationship between A & $\ddot{a}$

$$A_{[62]:\overline{3}|} = 1 - d\, \ddot{a}_{[62]:\overline{3}|}$$

$$= \underbrace{1 - (1 - (1/1.04))(2.571653)}_{0.9010903}$$

Same answer!

Question No. 2:

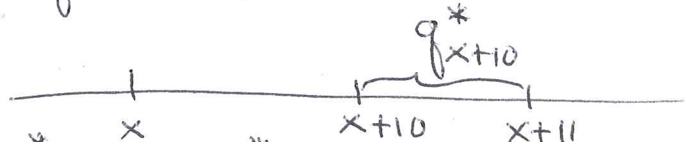
You are given:

- $q_{x+10} = 0.0083$
- $A_{x+10} = 0.5148$
- ${}_{10}E_x = 0.7055$
- $i = 3\%$

If q_{x+10} is increased by a constant 0.0010 and everything else remains, how much will A_x increase by?

Let $A_x^* =$ new A_x when $q_{x+10}^* = q_{x+10} + .001$

Using recursions,



$$A_x^* = A^{\,1}_{x:\overline{10|}} + {}_{10}E_x \left[v \underbrace{q_{x+10}^*}_{(q_{x+10} + .001)} + v \underbrace{p_{x+10}^*}_{(1 - q_{x+10} - .001) = p_{x+10} - .001} A_{x+11} \right]$$

$$= \underbrace{A^{\,1}_{x:\overline{10|}} + {}_{10}E_x \left[v q_{x+10} + v p_{x+10} A_{x+11} \right]}_{\text{old } A_x} + {}_{10}E_x \left[v(.001) - v(.001) A_{x+11} \right]$$

Thus, the increase is

$$A_x^* - A_x = {}_{10}E_x \left[v(.001) - v(.001) A_{x+11} \right]$$

${}_{10}E_x = .7055$

$v = 1/1.03$

$A_{x+10} = v q_{x+10} + v p_{x+10} A_{x+11}$

$$\Rightarrow A_{x+11} = \frac{A_{x+10} - v q_{x+10}}{v p_{x+10}}$$

$$= \frac{.5148 - (1.03)^{-1}(.0083)}{(1.03)^{-1}(1 - .0083)}$$

$$= .5263124$$

$$= \underbrace{.7055 \left[(1.03)^{-1} (.001) \right] \left[1 - .5263124 \right]}$$

$$= \underline{\underline{.000324453}}$$

Question No. 3:

For a whole life insurance issued to (50), you are given:

- A benefit of 100 is paid at the end of the year of death.
- Z is the present value random variable for this insurance.
- $\ddot{a}_{50} = 14.64$
- ${}^2A_{51} = 0.1321$
- $q_{50} = 0.0059$
- $i = 5\%$

Calculate Var[Z].

$Z = 100\,v^{K+1}$, $K = 0, 1, 2, \ldots$

K = curtate future lifetime of (50)

$$Var[Z] = 100^2\left[{}^2A_{50} - (A_{50})^2\right]$$

$$A_{50} = 1 - d\,\ddot{a}_{50} = 1 - (1 - 1/1.05)(14.64)$$

$$= .3028571$$

$${}^2A_{50} = v^2 q_{50} + v^2 p_{50}\,{}^2A_{51}$$

$$= (1.05)^{-2}\left[.0059 + (1-.0059)(.1321)\right]$$

$$= .1244631$$

$$= 100^2\left[.1244631 - (.3028571)^2\right]$$

$$= \underline{\underline{327.4069}}$$

Question No. 4:

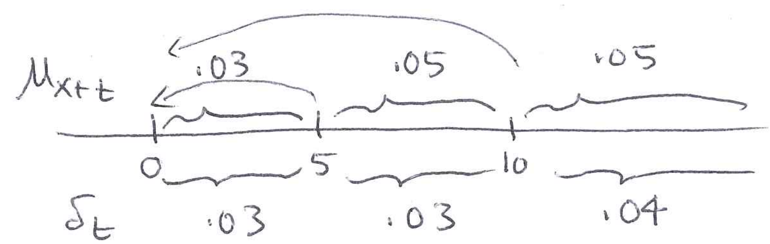
For a whole life insurance of \$1 on (x) with benefits payable at the moment of death, you are given:

$$\delta_t = \begin{cases} 0.03, & 0 < t \leq 10 \\ 0.04, & t > 10 \end{cases}$$

and

$$\mu_{x+t} = \begin{cases} 0.03, & 0 < t \leq 5 \\ 0.05, & t > 5 \end{cases}$$

Calculate the actuarial present value for this insurance.



$$APV(\text{insurance}) = \frac{.03}{.06}\left(1 - e^{-.06(5)}\right) + e^{-.06(5)} \frac{.05}{.08}\left(1 - e^{-.08(5)}\right) + e^{-.06(5)} e^{-.08(5)} \frac{.05}{.09}$$

$$= \underline{\underline{0.5581172}}$$

Question No. 5:

An insurance company sells whole life annuities to a group of 100 individuals each age 50. You are given:

- Their future lifetimes are independent.
- Mortality follows the `Illustrative Life Table` with $i = 6\%$.
- Each individual is paid 500 at the beginning of each year, if alive.

Using the Normal approximation, calculate the amount of fund needed at issue so that the company is 95% certain that there is enough money to pay the life annuities.

Let $Y_{agg} = 500(Y_1 + Y_2 + \dots + Y_{100})$ where Y_i = P.V.r.v. of a whole life annuity-due of \$1 on (50)

$$E[Y_{agg}] = 500 * 100\underbrace{E[Y_1]}_{\ddot{a}_{50}} = 500 * 100 * 13.2668 = 663{,}340$$

$$\underbrace{Var[Y_{agg}] = 500^2 * 100 \underbrace{Var[Y_1]}_{\frac{1}{d^2}\left[{}^2A_{50} - (A_{50})^2\right] = \frac{1}{(1-1/1.06)^2}\left[.09476 - (.24905)^2\right]}}_{255416889}$$

$$Pr[F \geq Y_{agg}] \approx Pr\left[Z \leq \underbrace{\frac{F - E[Y_{agg}]}{\sqrt{Var[Y_{agg}]}}}_{1.645}\right] = 0.95$$

1.645 → 95th percentile of a standard Normal

$$\therefore F = E[Y_{agg}] + 1.645\sqrt{Var[Y_{agg}]}$$
$$= 689{,}630$$

On a per policy basis, insurer would need 6,896.30

Question No. 6:

For a 3-year temporary life annuity-due on (45), you are given $v = 0.9$ and

k	payment	p_{45+k}
0	1	0.90
1	4	0.85
2	8	0.80

Calculate the variance of the present value of the indicated payments.

Let Y = p.v. of the annuity payments

k	$\Pr[K=k] = {}_kp_{45}\, q_{45+k}$	y	$y \cdot \Pr[K=k]$
0	0.100	1	0.100
1	$0.90(0.15) = 0.135$	$1+4v = 4.6$	0.6210
≥ 2	.765	$1+4v+8v^2 = 11.08$	8.4762
		sum	9.1972

y^2	$y^2 \Pr[K=k]$
1	0.100
21.16	2.8566
122.7664	93.9163
sum	96.8729

$$\mathrm{Var}[Y] = 96.8729 - (9.1972)$$

$$= \underline{\underline{12.28441}}$$

Question No. 7:

For a life annuity-due issued to (x):

- K denotes the curtate future lifetime of (x).
- The first payment does not start until the beginning of the 5th year.
- If (x) survives to reach the beginning of the 5th year, the first 10 payments of 1 each year is guaranteed for 10 years.
- A payment of 1 each year is made starting at the beginning of the 15th year so long as (x) is alive.

Which of the following gives the present value random variable for this life annuity?

(a) $\ddot{a}_{\overline{10}|} I(K < 15) + \ddot{a}_{\overline{K+1}|} I(K \geq 15)$

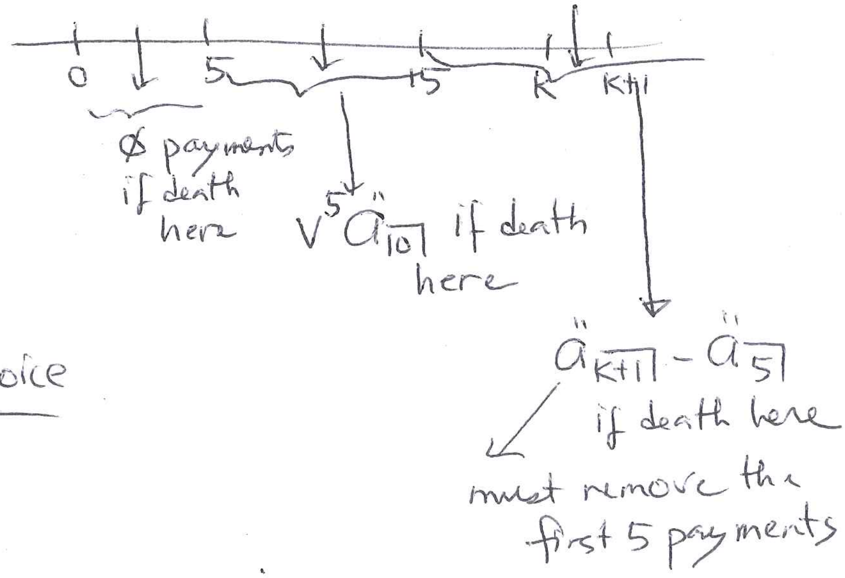
(b) $v^5\, \ddot{a}_{\overline{10}|} I(K < 15) + \ddot{a}_{\overline{K+1}|} I(K \geq 15)$

(c) $\ddot{a}_{\overline{10}|} I(5 \leq K < 15) + \ddot{a}_{\overline{K+1}|} I(K \geq 15)$

(d) $v^5 \left[\ddot{a}_{\overline{10}|} I(5 \leq K < 15) + \ddot{a}_{\overline{K+1}|} I(K \geq 15)\right]$

(e) $v^5\, \ddot{a}_{\overline{10}|} I(5 \leq K < 15) + (\ddot{a}_{\overline{K+1}|} - \ddot{a}_{\overline{5}|}) I(K \geq 15)$

This annuity is something of a 5 year deferred with a 10-year guarantee



Thus, (e) is the correct choice

Question No. 8:

For a two-year term life insurance policy issued to (x), you are given:

- A benefit of \$1 is payable at the end of the year of death.
- Z is the present value random variable for this insurance.
- $i = 0\%$
- $q_x = 0.60$
- $\text{Var}[Z] = 0.09$

Calculate q_{x+1}.

Since $i = 0\%$, $v = 1$

$$E[Z] = \cancel{v}\, q_x + \cancel{v^2}\, p_x\, q_{x+1} = .60 + .40\, q_{x+1}$$

$$E[Z^2] = \underbrace{\cancel{v^2}\, q_x + \cancel{v^4}\, p_x\, q_{x+1}}_{\text{evaluated @ } 2\delta} = .60 + .40\, q_{x+1}$$

$$\text{Var}[Z] = E[Z^2] - E[Z]^2$$

$$= .60 + .40\, q_{x+1} - (.60 + .40\, q_{x+1})^2 = .09$$

Let $w = .60 + .40\, q_{x+1}$

We solve a quadratic equation: $w^2 - w + .09 = 0$ $\quad (w - .9)(w - .1) = 0$

$$\cancel{w = \frac{1 \pm \sqrt{1 - 4(1)(.09)}}{2}}$$

$$= .90 \text{ or } \cancel{1.083095}\ .10$$

If $w = .90 \Rightarrow q_{x+1} = \dfrac{.90 - .60}{.40} = \cancel{0.207738}\ 0.75$ ~~Not possible~~

If $w = 1.083095 \Rightarrow q_{x+1} = \dfrac{\cancel{1.083095}\ 0.10 - .60}{.40} = \cancel{0.78}\ -1.25$ Not possible

Thus, the correct answer is $q_{x+1} = 0.75$

Question No. 9:

Leo is currently age 50 who purchases a whole life annuity-due policy which will pay him quarterly the following benefits:

- \$75,000 each year for the next 15 years and
- \$125,000 each year thereafter.

You are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 6\%$
- Deaths are uniformly distributed between integral ages.

Calculate the actuarial present value of Leo's life annuity benefits.

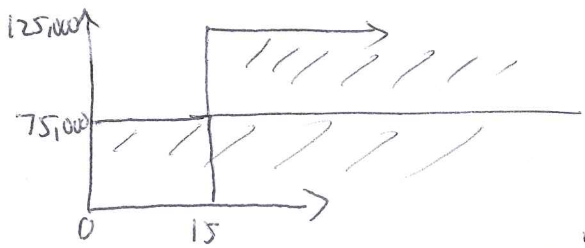
$i = .06$

$i^{(4)} = 4[1.06^{1/4} - 1]$

$= .05869538$

$d^{(4)} = 4[1 - 1.06^{-1/4}]$

$= .05784655$



$APV(\text{annuity}) = 75{,}000\, \ddot{a}_{50}^{(4)} + 50{,}000\ {}_{15}E_{50}\, \ddot{a}_{65}^{(4)}$

use relationship of $\ddot{a}^{(4)} \Leftrightarrow A^{(4)}$

$A_{50}^{(4)} = \frac{i}{i^{(4)}} A_{50} = \frac{.06}{.05869538}(.24905) = .2545856 \Rightarrow \ddot{a}_{50}^{(4)} = \frac{1 - A_{50}^{(4)}}{d^{(4)}}$

$= 12.88606$

Similarly

$A_{65}^{(4)} = \frac{i}{i^{(4)}} A_{65} = \frac{.06}{.05869538}(.43980) = .4495754 \Rightarrow \ddot{a}_{65}^{(4)} = \frac{1 - A_{65}^{(4)}}{d^{(4)}}$

$= 9.515253$

${}_{15}E_{50} = {}_{5}E_{50}\ {}_{10}E_{55} = (.72137)(.48686) = .3512062$

$APV(\text{annuity}) = 25{,}000\left[3(12.88606) + 2(.3512062)(9.515253)\right]$

$= \underline{\underline{1{,}133{,}546}}$

Question No. 10:

Let Z denote the present value random variable for a 20-year pure endowment policy to (45) with benefit of \$1,000. You are given:

- ${}_5p_{45} = 0.98$
- ${}_{10}p_{50} = 0.92$
- ${}_5p_{60} = 0.90$
- $i = 5\%$

Calculate Var[Z].

$Z = 1000\, v^{20}\, I(T \geq 20)$

where T = future lifetime of (45)

$I(T \geq 20) \sim$ Bernoulli with

$p = E[I(T \geq 20)] = {}_{20}p_{45}$

$= {}_5p_{45} * {}_{10}p_{50} * {}_5p_{60}$

$= .98(.92)(.90)$

$= .81144$

$Var[Z] = 1000^2\, v^{40}\, (.81144)(1 - .81144)$

$= 1000^2 (1.05)^{-40} (.81144)(1 - .81144)$

$= \underline{\underline{21,733.72}}$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK