MATH 3630

Actuarial Mathematics I

Sample Test 2

Time Allowed: 1 hour

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTION Student ID: Emil

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

For a three-year term life insurance on (x), you are given:

- Z is the present value random variable for the death benefits;
- death benefits are payable at the end of the year of death;
- discount rate $i = 5\%$; and

k	b_{k+1}	q_{x+k}
0	10	0.01
1	5	0.02
2	1	0.04

Calculate E(Z).

$$E(Z) = \sum_{K=0}^{2} b_{K+1} V^{K+1} {}_{K}P_x \, q_{x+K}$$

$$= 10 v q_x + 5 V^2 P_x q_{x+1} + 1 V^3 {}_2P_x q_{x+2}$$

$$= 10 \frac{1}{1.05} 0.01 + 5 \frac{1}{1.05^2} .99(.02) + \frac{1}{1.05^3} .99(.98)(.04)$$

$$= \underline{\underline{.2186}}$$

Question No. 2:

Denote by Y the present value random variable for a whole life annuity-due on (x). You are given that $v = 0.8$ and

$$q_{x+k} = 0.1, \quad \text{for all } k = 0, 1, 2, \cdots.$$

Calculate the expected value of Y.

Using "Current Payment Technique",

$$E(Y) = \ddot{a}_x = \sum_{k=0}^{\infty} v^k {}_kP_x$$

$$= \sum_{k=0}^{\infty} (.8)^k (.9)^k$$

$$= \sum_{k=0}^{\infty} (.72)^k = \frac{1}{1-.72} = \underline{3.5714}$$

Question No. 3:

Cindy is currently age 35. Her mortality follows DeMoivre's law with $\omega = 120$.

She purchases a whole life insurance policy that pays a benefit of 1,000,000 at the moment of death.

Compute the actuarial present value of her death benefits if $i = 10\%$.

Let T_{35} be Cindy's future lifetime. Then, $T_{35} \sim$ Uniform on $(0, 85)$

$$\text{APV(benefits)} = 1000000 \bar{A}_{35}$$

$$= 1000000 \int_0^{85} (1.10)^{-t} \frac{1}{85} dt$$

$$= \frac{1000000}{85} \left[\frac{1}{-\log 1.1} \left((1.10)^{-85} - 1 \right) \right]$$

$$= \underline{\underline{123,398.57}}$$

Question No. 4:

For a special type of whole life insurance issued to (30), you are given:

- death benefits are 1,000 for the first 10 years and 5,000 thereafter;
- death benefits are payable at the moment of death;
- deaths are uniformly distributed over each year of age interval;
- $i = 5\%$; and
- the following table of actuarial present values:

x	$1000A_x$	$1000\,{}_5E_x$
30	112.31	779.79
35	138.72	779.20
40	171.93	777.14

Calculate the actuarial present value of the benefits for this policy.

$$\text{APV(benefits)} = 1000\,\bar{A}_{30} + 4000\,\underbrace{{}_{10}E_{30}}_{\downarrow}\,\bar{A}_{40}$$

by UDD
$$= 1000\,\frac{i}{\delta}A_{30} + 4000\ {}_5E_{30}\,{}_5E_{35}\cdot\frac{i}{\delta}A_{40}$$

$$= \frac{.05}{\log 1.05}\left(112.31 + 4(171.93)(.77979)(.7792)\right)$$

$$= \underline{\underline{543.33}}$$

Question No. 5:

In a club of 100 membership all age x, the members decided to each contribute an amount of G to a fund which will pay 1,000 at the moment of death of each member.

You are given:

- the future lifetimes of the members are independent;
- $i = 10\%$, $\bar{A}_x = 0.06$, and ${}^2\bar{A}_x = 0.01$; and
- the members want the total contributions to be sufficient to pay the club's promised obligations with probability 0.95.

Using Normal approximation, calculate G. (Note that the 95th percentile of a standard Normal is 1.645.)

$Z =$ PV of benefits random variable

$$= 1000 \sum_{i=1}^{100} Z_i, \text{ where } Z_i = v^{T}, \text{ for all } i$$

$$E(Z_i) = E(v^T) = \bar{A}_x = .06$$

$$Var(Z_i) = {}^2\bar{A}_x - \bar{A}_x^2 = .01 - (.06)^2 = .0064$$

$$E(Z) = 1000(100)(.06) = 6000$$

$$Var(Z) = 1000^2(100)(.0064) = 0.64 \times 1000^2$$

$$\therefore P(Z \le 100G) = P\left(\frac{Z - E(Z)}{\sqrt{Var(Z)}} \le \underbrace{\frac{100G - 6000}{\sqrt{0.64 \times 1000^2}}}_{1.645}\right) = .95$$

$$\therefore 100G = 6000 + 1.645\sqrt{0.64 \times 1000^2}$$

$$= 7316 \Rightarrow G = 73.16$$

EW 12-11-08

Question No. 6:

A person age 40 wins 100,000 in an actuarial lottery. Rather than receiving the money at once, the winner is offered the actuarially equivalent option of receiving an annual payment of H (at the beginning of each year) guaranteed for 10 years and continuing thereafter for life.

You are given that interest rate $i = 4\%$ and the following values extracted from a mortality table:

- $A_{40} = 0.23056$;
- $A_{50} = 0.32907$; and
- $A^{\,1}_{40:\overline{10}|} = 0.01151$.

Calculate the value of H.

By actuarial equivalence, we have

$$100{,}000 = H\ddot{a}_{\overline{10}|} + H\,{}_{10|}\ddot{a}_{40} = H\ddot{a}_{\overline{10}|} + H\,{}_{10}E_{40}\,\ddot{a}_{50}$$

So that $H = \dfrac{100{,}000}{\ddot{a}_{\overline{10}|} + {}_{10}E_{40}\,\ddot{a}_{50}}$.

We know that

$$\ddot{a}_{\overline{10}|} = \frac{1-v^{10}}{d} = 8.435332$$

and that

$$\ddot{a}_{50} = \frac{1-A_{50}}{d} = \frac{1-0.32907}{.04/1.04} = 17.44418.$$

Using the relationship, $A_{40} = A^{\,1}_{40:\overline{10}|} + {}_{10}E_{40}\,A_{50}$

we get $${}_{10}E_{40} = \frac{A_{40} - A^{\,1}_{40:\overline{10}|}}{A_{50}} = \frac{.23056 - .01151}{.32907} = .665664$$

$$\therefore H = \frac{100{,}000}{8.435332 + .665664(17.44418)} = \underline{\underline{4{,}988.21}}$$

Question No. 7:

For a whole life insurance of 1,000 on (x) with benefit payable at the moment of death, you are given:

$$\delta_t = 0.05, \quad \text{for all } t > 0,$$

and

$$\mu_{x+t} = \begin{cases} 0.006, & 0 < t \leq 10 \\ 0.007, & t > 10 \end{cases} .$$

Calculate the actuarial present value for this insurance.

$$APV(\text{insurance}) = 1000\,\bar{A}_x = 1000\int_0^\infty v^t \, {}_tp_x \, \mu_{x+t}\,dt$$

$$= 1000\left[\int_0^{10} e^{-.05t} e^{-.006t} \, .006\,dt + \int_{10}^\infty e^{-.05t} e^{-.006(10)} e^{-.007(t-10)} \, .007\,dt\right]$$

$$= 1000\left[\frac{.006}{.056}\left(1-e^{-.56}\right) + \frac{.007e^{.01}}{.057}\underbrace{\int_{10}^\infty .057e^{-.057t}\,dt}_{e^{-.57}}\right]$$

$$= \underline{\underline{116.09}}$$

Question No. 8:

For a continuous whole life insurance issued to (x), you are given:

- forces of mortality and interest are each constant; and
- $\mathrm{E}(v^{2T_x}) = \frac{1}{4}$ where T_x denotes the future lifetime of (x).

Calculate $\bar{A}_x$.

For constant forces of mortality and interest, we know

$$\bar{A}_x = \frac{\mu}{\mu+\delta} \quad \text{and} \quad \underbrace{E\left(v^{2T_x}\right) = {}^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = \frac{1}{4}}$$

this implies $4\mu - \mu = 2\delta$

$$\mu = \frac{2}{3}\delta$$

$$\therefore \bar{A}_x = \frac{\frac{2}{3}\delta}{\frac{2}{3}\delta + \delta} = \frac{2}{5} = \underline{\underline{0.4}}$$

Question No. 9:

Suppose you are given that:

- mortality follows the *Illustrative Life Table*; and
- $i = 6\%$.

Approximate the value of $\ddot{a}^{(2)}_{40:\overline{30|}}$ and interpret this value.

$$\ddot{a}^{(2)}_{40:\overline{30|}} = \frac{1 - A^{(2)}_{40:\overline{30|}}}{d^{(2)}} \quad \text{where } d^{(2)} = 2\left[1 - 1.06^{-1/2}\right] = .05742828$$

and

$$A^{(2)}_{40:\overline{30|}} = \frac{i}{i^{(2)}} A^{\,1}_{40:\overline{30|}} + {}_{30}E_{40} \quad \text{by UDD}$$

$$= \frac{i}{i^{(2)}}\left[A_{40} - {}_{30}E_{40}\, A_{70}\right] + {}_{30}E_{40}$$

$$= \frac{i}{i^{(2)}}\left[A_{40} - {}_{20}E_{40}\; {}_{10}E_{60}\, A_{70}\right] + {}_{30}E_{40}$$

$$= \underbrace{\frac{.06}{2\left[1.06^{1/2} - 1\right]}\left[.16132 - (.27414)(.45120)(.51495)\right] + (.27414)(.45120)}_{\cancel{.22834124}\quad 0.2227598}$$

$$\therefore\ \ddot{a}^{(2)}_{40:\overline{30|}} \approx \frac{1 - \cancel{.22834124}\ 0.2227598}{.05742828} = 13.53$$

provides for the APV of a 30-year temporary life annuity-due to (40) of \$1 per year payable every 6 months.

Question No. 10:

You are given:

- $A_x = 0.5263$;
- $\ddot{a}_{x+1} = 9.618$; and
- $i = 5\%$.

Calculate q_x.

By recursion, $\ddot{a}_x = 1 + v p_x \ddot{a}_{x+1}$

So that $p_x = \dfrac{(\ddot{a}_x - 1)(1+i)}{\ddot{a}_{x+1}}$

where $\ddot{a}_x = \dfrac{1-A_x}{d} = \dfrac{1-.5263}{.05/1.05} = 9.9477$

$\therefore p_x = \dfrac{(9.9477-1)(1.05)}{9.618} = .9768$

$q_x = 1 - p_x = 1 - .9768 = \underline{\underline{.0232}}$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK