

Michigan State University
STT 455 - Actuarial Models I
Sample Test 1
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____ Section No.: _____

- There are five (5) multiple choice (MC) and one (1) written-answer questions here and you are to answer all questions asked. Points assigned are clearly indicated on each question.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

MC Question No. 1: (10 points)

A class of species follows a mortality pattern described by

$$S_0(x) = \left(\frac{2}{2+x} \right)^2, \text{ for } x \geq 0,$$

where x is measured in years.

Calculate the average number of years yet to live for a one-year old member of this class of species.

- (a) 1.0
- (b) 1.5
- (c) 2.0
- (d) 2.5
- (e) 3.0

MC Question No. 2: (10 points)

You are given:

$$\mu_x = \begin{cases} 0.01, & 0 < x < 20 \\ 0.02, & x \geq 20 \end{cases}$$

Calculate ${}_8p_{15}$.

- (a) 0.852
- (b) 0.878
- (c) 0.896
- (d) 0.914
- (e) 0.923

MC Question No. 3: (10 points)

Suppose you are given the following select-and-ultimate mortality table:

$[x]$	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	l_{x+3}	$x+3$
20	35,600	35,591	35,583	35,572	23
21	35,587	35,576	35,562	35,553	24
22	35,574	35,561	35,547	35,534	25

Calculate the probability that a life with select age 20 will survive for two years but die the following two years.

- (a) 0.0001
- (b) 0.0004
- (c) 0.0008
- (d) 0.0012
- (e) 0.0015

MC Question No. 4: (10 points)

For a fixed age x , you are given the following probabilities:

- ${}_{20}p_x = 0.40$
- ${}_5p_x = 0.78$
- ${}_5p_{x+15} = 0.75$

Calculate ${}_{15}q_{x+5}$.

- (a) 0.2
- (b) 0.3
- (c) 0.4
- (d) 0.5
- (e) 0.6

Question No. 5: (10 points)

For a whole life insurance with benefits payable at the moment of death issued to (40), you are given:

- The death benefit at time t is $b_t = 10(1.01)^t$ for $t > 0$.
- Mortality follows deMoivre's law with $\omega = 100$.
- $\delta = 5\%$

Calculate the actuarial present value for this insurance.

- (a) 3.8
- (b) 4.1
- (c) 4.4
- (d) 4.7
- (e) 5.0

There are four (4) parts to the written-answer portion of this test. You are to answer all parts. Please provide as much details of your calculations as possible to get your partial points for any incorrect answers.

Let X be the lifetime (of a newborn) random variable with survival function defined by

$$S_0(x) = e^{-(x/\lambda)^k} \text{ for } x \geq 0,$$

where λ and k are both parameters.

- (i) (20 points) Give constraints on the values of the parameters λ and k so that the function above is a legitimate survival function. Justify your solution.

(ii) (10 points) Find the force of mortality at age x , μ_x

(iii) (5 points) Find an expression for the density function $f_0(x)$.

(iv) (15 points) Suppose $\lambda = \frac{15}{2}$ and $k = \frac{3}{4}$. Calculate ${}_{20|10}q_{20}$ and interpret this probability.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK