5 MC + 1 WA

50 50

Q1 $S_0(x) = \left(\frac{2}{2+x}\right)^2$, $x \geq 0$ average age for a 1 year old

$E[T_1] = \mathring{e}_1 = \int_0^\infty {}_tp_1 \, dt$

$\mathring{e}_x = \int_0^\infty {}_tp_x \, dt$

$\frac{S_0(x+t)}{S_0(x)}$

$\frac{S_0(1+t)}{S_0(1)} = \frac{\left(\frac{\cancel{2}}{2+1+t}\right)^2}{\left(\frac{\cancel{2}}{2+1}\right)^2} = \left(\frac{3}{3+t}\right)^2 = 9(3+t)^{-2}$

$= 9\int_0^\infty (3+t)^{-2} dt = 9 \left.\frac{(3+t)^{-1}}{-1}\right|_0^\infty = 9 \cdot \frac{1}{3}$

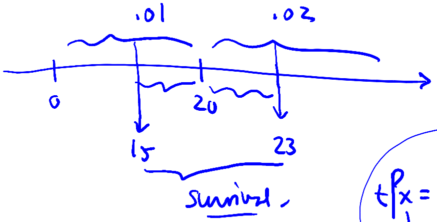
$= 3$

e

Q2

$$\mu_x = \begin{cases} .01, & 0 < x < 20 \\ .02, & x \geq 0 \end{cases}$$

$${}_{8}p_{15} = {}_{5}p_{15}\;{}_{3}p_{20}$$



$$= \underbrace{e^{-.01(5)}\; e^{-.02(3)}}$$

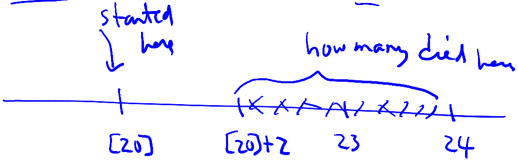
$${}_{t}p_{x} = e^{-\mu t}$$

$$e^{-.11} = \underline{.8958341}$$

Q3

3-year

l's



$${}_{2|2}q_{[20]} = \frac{l_{[20]+2} - l_{24}}{l_{[20]}} = \frac{35583 - 35553}{35600}$$

$$= .0008426966$$

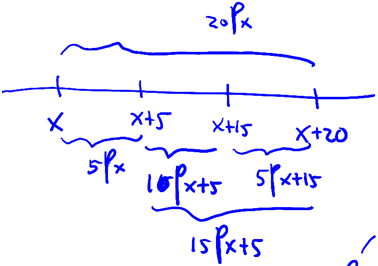
5

Q4

$_{20}p_x$ ✓

$_{5}p_x$ ✓

$_{5}p_{x+15}$



$$\frac{\overset{.40}{_{20}p_x}}{\underset{.78}{_{5}p_x}} = {}_{15}p_{x+5}$$

$$_{15}q_{x+5} = 1 - {}_{15}p_{x+5}$$

$$= 1 - \frac{.40}{.78} = \underline{0.4871795}$$

Q5

$$b_t = 10(1.01)^t$$

$$v^t = e^{-\delta t} = e^{-.05t}$$

$$\int a^t\,dt = \frac{a^t}{\log(a)}$$

de Moivres $\omega = 100$

(40)

$T_{40} \sim$ de Moivre's $\left(0, \underbrace{100-40}_{60}\right)$

$$APV = \int_0^{\infty} b_t\, v^t\, f_T(t)\,dt$$

$10(1.01)^t$

$e^{-.05t}$

$$f_T(t) = \frac{1}{60}, \quad 0 \le t \le 60$$

$$\frac{10}{60}\int_0^{60} \left(1.01\,e^{-.05}\right)^t dt$$

$$= \frac{10}{60}\left[\frac{\left(1.01e^{-.05}\right)^t}{\log\left(1.01e^{-.05}\right)}\Bigg|_0^{60}\right] = \underline{\underline{3.7851}},$$

(a)

Written Answer

$$\underbrace{S_0(x) = e^{-\left(x/\lambda\right)^{k}}, \quad x \geq 0}$$

(i) non-increasing: $\frac{d}{dx} S_0(x) = \underbrace{e^{-\left(x/\lambda\right)^{k}} \cdot -k\left(\frac{x}{\lambda}\right)^{k-1} \cdot \frac{1}{\lambda}}_{\leq 0}$

$k > 0$

$\lambda > 0$

$k = 0 \Longrightarrow S_0(x) = e^{-1}$ not legitimate

$S_0(0) = e^{0} = 1$

$S_0(\infty) = e^{-\infty} \to 0$

(ii) $\mu_x = -\dfrac{d\,S_0(x)}{dx}\left(\dfrac{1}{S_0(x)}\right)$ must know

$S_0(x) = e^{-\left(\frac{x}{\lambda}\right)^k}$

$= \dfrac{\cancel{-}1}{\cancel{e^{-(x/\lambda)^k}}}\,\cancel{e^{-\left(\frac{x}{\lambda}\right)^k}}\left(\cancel{-}k\left(\frac{x}{\lambda}\right)^{k-1}\frac{1}{\lambda}\right)$

$\lambda > 0$ -

$k > 0$ -

$= \dfrac{k}{\lambda}\left(\dfrac{x}{\lambda}\right)^{k-1}$

(iii) $f_0(x) =$

$$+\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(x/\lambda\right)^k}$$

$$\mu_x = \frac{f_0(x)}{S_0(x)}$$

$$f_0(x) = \mu_x S_0(x)$$

$$= -\frac{d}{dx} S_0(x)$$

(iv) $\lambda = \frac{15}{2}$ $\quad k = 3/4$

$$S_0(x) = e^{-\left(\frac{x}{\lambda}\right)^k}$$

$$= e^{-\left(\frac{2x}{15}\right)^{3/4}}$$

$$S_x(t) = \frac{S_0(x+t)}{S_0(x)}$$

$${}_{20|10}q_{20} = S_{20}(20) - S_{20}(30)$$

20 $\quad$ 40 $\quad$ 50

$$= \frac{S_0(40) - S_0(50)}{S_0(20)}$$

$$= \frac{e^{-\left(\frac{80}{15}\right)^{.75}} - e^{-\left(\frac{100}{15}\right)^{.75}}}{e^{-\left(\frac{40}{15}\right)^{.75}}} = \underline{\underline{.1138642}}$$

$_tp_x$ - interpretation of
$_tq_x$ - these probabilities
$_{m|n}q_x$ -

MC -

- interpolation between integral ages

UDD, CF

- other types of insurance

APV
PV random variable
moment @ 2δ
$\Leftarrow$
term, pure endowment
whole life, deferred,
endowment