MATH 3630
Actuarial Mathematics I
Sample Test 1
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

The force of mortality for a newborn is given by

$$\mu_x = \frac{1}{460 - cx}, \quad \text{for} \quad 0 \leq x < \omega,$$

for some positive constant c.

You are also given: $\mathring{e}_{35} = 64$

Calculate ${}_{10}q_{35}$ and interpret this value.

Question No. 2:

You are given:

- The probability that (50) dies before reaching age 65 is 0.117.
- The probability that (50) dies between the ages of 65 and 75 is 0.185.
- $\ell_{75} = 65310$

Calculate ℓ_{50}.

Question No. 3:

Assume the constant force of mortality assumption holds between integral ages. You are given:

$S_0(65) = 0.730$

$S_0(66) = 0.725$

$S_0(67) = 0.720$

Calculate the probability that a person who is currently age 65 years and 4 months will survive at least one more year.

Question No. 4:

You are given:

- $\ell_x = 100(\omega - x)^{1/2}$, for $0 \le x \le \omega$
- $\mathring{e}_{40} = 40$
- T_{40} is the future lifetime random variable for a person currently age 40.

Calculate $\mathrm{Var}[T_{40}]$.

Question No. 5:

Assume mortality follows the *Illustrative Life Table*.

Suppose that Uniform Distribution of Death (UDD) assumption holds between integral ages.

Calculate ${}_{2.25|1.5}q_{30}$ and interpret this probability.

Question No. 6:

The mortality for Joshua, age 30, follows de Moivre's law with $\omega = 105$.

In the coming year, Joshua intends to join an expedition for mountain climbing. If he does, his assumed mortality will be adjusted so that for the coming year only, he will have a force of mortality increased by 50%.

Calculate the decrease in his 5-year temporary complete life expectation if he pursues this expedition.

Question No. 7:

For a certain population, you are given:

- The mortality for males follows a Gompertz law with $B = 9.0 \times 10^{-5}$ and $c = 1.09$.
- The mortality for females follows a Gompertz law with $B = 4.5 \times 10^{-5}$ and $c = 1.10$.
- Immediately after 30 years from birth, 40% are males and 60% are females.

Calculate the probability that a randomly chosen 30-year-old from this population will die within the next 10 years.

Question No. 8:

For a given life age 40, it is estimated that an impact of a medical breakthrough will be an increase of 3.2 years in $\mathring{e}_{40}$.

Prior to the medical breakthrough, mortality followed a Generalized de Moivre's law with $\alpha = 0.5$ and limiting age $\omega = 105$.

After the medical breakthrough, a Generalized de Moivre's law for mortality still applies with the same parameter α, but with a different limiting age.

Calculate the new limiting age as a result of the medical breakthrough.

Question No. 9:

Suppose you are given the following select-and-ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	x+3
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

Calculate the probability that a life with select age 61 will survive for two years but die the following three years.

Question No. 10:

The force of mortality for a substandard life (x) is expressed as

$$\mu^*_{x+t} = \mu_{x+t} + k,$$

for some positive constant k, where μ_{x+t} is the force of mortality of a standard life (x).
You are given:

- The probability that a standard life (x) survives another year is 0.80.
- The probability that a substandard life (x) will die within the following year is 0.34.

Calculate k.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK