MATH 3630
Actuarial Mathematics I
Sample Test 1
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____ **Student ID:** _____

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

Let X be the age-at-death random variable.

Assume X follows deMoivre's law with $\omega = 100$.

Calculate ${}_{10}m_{20}$.

Question No. 2:

Assume that mortality follows the *Illustrative Life Table*.

Calculate the probability that a life (65) will die between ages 80 and 90.

Question No. 3:

You are given:

$$ {}_{k|}q_0 = 0.1(k+1), \quad \text{for } k = 0, 1, 2 \text{ and } 3. $$

Suppose UDD holds between integral ages.

Compute the value of ${}_{2.75}p_0$ and interpret this probability.

Question No. 4:

You are given:

$$\begin{aligned} {}_5p_{40} &= \frac{4}{5} \\ {}_{10}p_{45} &= \frac{3}{5} \\ {}_{10}p_{55} &= \frac{2}{5} \end{aligned}$$

Find the probability that a life (40) will die between ages 55 and 65.

Question No. 5:

Suppose you are given the survival function:

$$S_X(x) = \left(1 - \frac{x}{\omega}\right)^{\alpha}, \text{ for } 0 \leq x \leq \omega.$$

Prove the following: $\mu_x \cdot \mathring{e}_x = \dfrac{\alpha}{\alpha + 1}$.

Question No. 6:

Suppose that for an 80-year-old fellow, his force of mortality is given by

$$\mu_{80+t} = \frac{1}{10-t}, \text{ for } 0 \leq t < 10.$$

Calculate the probability that this fellow will die between ages 85 and 90.

Question No. 7:

The following is an extract from a *standard* mortality table:

x	q_x
40	.00278
41	.00298
42	.00320

A *substandard* table is obtained from this *standard* table by adding a constant $c = 0.10$ to the force of mortality. This results in mortality rates denoted by q_x^s, with the superscript s denoting *substandard*.

Calculate the probability that a *substandard* life (40) will die between ages 41 and 42.

Question No. 8:

Suppose you are given the following select-and-ultimate mortality table:

x	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}
95	300	60	15
96	175	10	0
97	15	0	0
98	1	0	0

Assuming UDD between integral ages, calculate ${}_{2|0.5}q_{[95]}$.

Question No. 9:

Suppose you are given that:

$$\ell_x = 1000\left(27 - \frac{3}{10}x\right)^{1/3}, \text{ for } 0 \leq x \leq 90.$$

Calculate the average future lifetime of a newborn.

Question No. 10:

You are given the survival function:

$$S_X(x) = \left(1 - \frac{x}{\omega}\right)^{5/2}, \text{ for } 0 \leq x \leq \omega.$$

If $\mu_{80} = 0.05$, calculate $\mathring{e}_{60:\overline{25}|}$ and interpret this value.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK