(1) Gross Premium - G

10MC

Chap. 6

5

2, 3, 4, 5

2

$$APVFP = APVFB + \underbrace{APVFE}$$

$$G\ddot{a}_{45} \qquad 100000\,A_{45}$$

$$P = \frac{100000\,A_{45}}{\ddot{a}_{45}}$$

$$.15G + .05G\ddot{a}_{45}$$

$$+ 45 + 10\ddot{a}_{45}$$

$$G(\cancel{.95\ddot{a}_{45}} - .15) = \frac{100000\,\overset{.20120}{A_{45}} + 45 + 10\,\overset{14.1121}{\ddot{a}_{45}}}{.95\,\underbrace{\ddot{a}_{45}}_{14.1121} - .15}$$

$$G = 1531.787$$

② π = premium

$$\frac{\mu}{\mu+\delta} \qquad \frac{1}{\mu+\delta}\left(1-e^{-(\mu+\delta)n}\right)$$

$$\cancel{\bar{A}_x = 1-\delta\,\bar{a}_x}$$

$\mu = .01$

0 — 10

δ: $\underbrace{.04}$ $\underbrace{.05}$

$$APVFP = \pi\,\bar{a}_x$$

$$APVFB = 100\,\bar{A}_x \qquad \frac{\mu}{\mu+\delta}$$

$$\underbrace{\frac{\mu}{\mu+\delta}\left(1-e^{-(\mu+\delta)n}\right)}$$

$$\bar{A}_x = \frac{.01}{.05}\left(1-e^{-.05(10)}\right) + \underbrace{e^{-(.04+.01)(10)}}\frac{.01}{.06} = .1797823$$

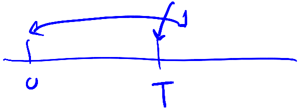
$$\bar{a}_x = \frac{1}{.05}\left(1-e^{-.5}\right) + e^{-.5}\frac{1}{.06} = 17.97823$$

$$\pi = \frac{100\,\bar{A}_x}{\bar{a}_x} = \frac{100(.1797823)}{17.97823} = \cancel{0.01 = \mu} \qquad 100(.01) = \underline{100\mu}$$

$$\bar{A}_x = \int_0^\infty v^t \cdot {}_tp_x \, \mu_{x+t} \, dt = \mu \text{ if constant}$$

$$\bar{a}_x \hookrightarrow \int_0^\infty v^t \, {}_tp_x \, dt$$

(3) fully continuous



$$L_0^g = PVFB_0 + PVFE_0 - PVFP_0$$

$1 \cdot v^T$

$.02 + .005\,\bar{a}_{\overline{T|}}$

$\frac{1-v^T}{\delta}$

$-\pi\,\bar{a}_{\overline{T|}}$

$\frac{1-v^T}{\delta}$

$$= \left(1 - \frac{.005}{\delta} + \frac{\pi}{\delta}\right) v^T + \left(.02 + \frac{.005}{\delta} - \frac{\pi}{\delta}\right)$$

$$Var(v^T) = {}^2\bar{A}_x - (\bar{A}_x)^2$$

$.3 \qquad (.5)^2$

$$Var[L_0^g] = \left(1 - \frac{.005}{\delta} + \frac{\pi}{\delta}\right)^2 [.05]$$

$\delta = .05 \qquad \delta = .05$

$$= \left(1 - \frac{.005}{.05} + \frac{.057}{.05}\right)^2 (.05) = \underline{\underline{0.20808}}$$

$$\pi = \frac{\bar{A}_x}{\bar{a}_x + .007} = \frac{\bar{A}_x}{\frac{1-\bar{A}_x}{\delta} + .007} = \frac{.5}{\frac{1-.5}{.05} + .007} = .05 + .007$$

$$= .057$$

$$\underbrace{Var[L_0^S] = \left(1 - \frac{.005}{.05} + \frac{.057}{.05}\right)^2 (.05)}_{\approx ?}$$

(4)

$q_x = .05$ 2-year term

$q_{x+1} = .08$ 10

P = premium

$\text{APV FP} = \text{APV FB}$ $\Leftarrow$ Equivalence Principle

$$P + P v p_x = 10\left(v q_x + v^2 p_x q_{x+1}\right)$$

$q_x = .05$, $p_x = .95$, $q_{x+1} = .08$

$$P(1 + v p_x), \quad 1 + v = \frac{1}{.05}, \quad p_x = .95$$

net premium

$$P = \frac{1.165533}{1.904762} = \underline{\underline{.6119048}}$$

π — P no expenses — benefit premium or net premium

π — G with expenses — gross premium

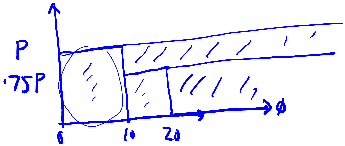
$$\text{APVFP} = \text{APVFB} + \text{APVFE}$$

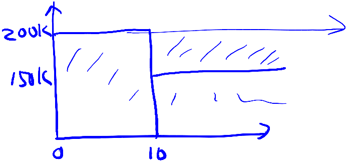
$$P(1 + v\,p_x) = 10\left(v\,q_x + v^2\,p_x\,q_{x+1}\right) + .40 + .10\,v\,p_x$$

$$P = \frac{10\left(v\,q_x + v^2\,p_x\,q_{x+1}\right) + .40 + .10\,v\,p_x}{1 + v\,p_x}$$

$$P = \frac{1.656009}{1.904762} = \underline{\underline{0.8694048}}$$

5 APVFP = APVFB





$$100{,}000\left[2A_{50} - .5\,{}_{10}E_{50}A_{60}\right]$$

$$P\ddot{a}_{50} - .25P\,{}_{10}E_{50}\,\ddot{a}_{60} - .75P\,{}_{20}E_{50}\,\ddot{a}_{70}$$

.24905 .51081 .36913

$$P = \frac{100{,}000\left(2A_{50} - .5\,{}_{10}E_{50}A_{60}\right)}{\ddot{a}_{50} - .25\,{}_{10}E_{50}\,\ddot{a}_{60} - .75\,{}_{20}E_{50}\,\ddot{a}_{70}} = \frac{40{,}382.24}{10.36228}$$

13.2668 11.1454 .23047 8.5693

$\cancel{3{,}897.0422}$

⑥ $\Pr[L_0 > 200] \iff \Pr[K \gtrless m]$

ILT
6%

discrete

$P = 11.35$

$K \gtrless$

$$\underbrace{L_0 = 1000 v^{K+1} - 11.35\, \ddot{a}_{\overline{K+1|}} > 200}$$

$$\ddot{a}_{\overline{K+1|}} = \frac{1 - v^{K+1}}{d}$$

$$\left(1000 + \frac{11.35}{d}\right) v^{K+1} - \frac{11.35}{d} > 200$$

$$v^{K+1} > \frac{200 + 11.35/d}{1000 + 11.35/d}$$

$$\underbrace{K+1}\,(\underbrace{\log v}_{-\delta}) > \log(\ldots\ldots)$$

$$d = \frac{.06}{1.06}$$

$$K\cancel{+1} < \underbrace{-\frac{1}{\delta}\log(\ldots\ldots) - 1}$$

$$\delta = \log(1.06)$$

$L_0 > 200$

$$K < 17.83941 \iff K \le 17$$

$$\Pr[L_0 > 200] = \Pr[K \le 17] = {}_{18}q_{40}$$

$$= 1 - {}_{18}p_{40}$$

$$= 1 - \frac{l_{58}}{l_{40}}$$

$$= 1 - \frac{8389826}{9313166} = .09914351$$

$$\underbrace{\Pr[K \le k] = {}_{k+1}q_x}$$

Woolhouse's approximation