Michigan State University

STT 455 - Actuarial Models I

Final Examination

Tuesday, 10 December 2013 5:45 - 7:45 PM

Total Score: 100 points

Name: Suggested Solutions **Section 2**

- There are ten (10) multiple choice questions here and you are to answer all questions asked. Each question is worth 10 points.
- Please double check your work as no partial points will be granted.
- Please write legibly.
- The Illustrative Life Table (ILT) is attached in the last two pages of this paper.
- Anyone caught writing after time has expired will be given a mark of zero.
- Good luck.
- Have a Happy and Healthy Christmas and New Year!

Question	Worth	Score
1	10	C
2	10	B
3	10	E
4	10	B
5	10	Defective
6	10	C
7	10	B
8	10	E
9	10	C
10	10	C
Total	100	

Question No. 1: (10 points)

For a fully continuous whole life insurance of \$1 issued to (40), you are given:

- Mortality follows De Moivre's (or Uniform distribution) law with $\omega = 100$.
- $\delta = 0.05$
- Premium, based on the Equivalence Principle, is paid continuously at the annual rate of P.

Calculate P.

(a) 0.015

(b) 0.021

(c) 0.023

(d) 0.025

(e) 0.031

$$APV(FP_0) = APV(FB_0)$$

$$P\bar{a}_{40} = \bar{A}_{40} \Rightarrow P = \bar{A}_{40}/\bar{a}_{40}$$

When you have De Moivre's, it is easier to work with insurances. So, we have

$$\bar{A}_{40} = \int_0^{60} e^{-.05t} \frac{1}{60} dt = \frac{1}{60}\frac{1}{.05}\left(1-e^{-.05(60)}\right)$$

$$= \frac{1}{3}(1-e^{-3}) = 0.3167376$$

$$P = \frac{\bar{A}_{40}}{\bar{a}_{40}} = \frac{\bar{A}_{40}}{(1-\bar{A}_{40})/\delta} = \delta\frac{\bar{A}_{40}}{1-\bar{A}_{40}}$$

$$= .05\frac{0.3167376}{1-0.3167376}$$

C

$$= \underline{0.23}17833$$

Question No. 2: (10 points)

A fully discrete whole life insurance of \$100 is issued to (46). You are given:

- Expenses consist of 10% of annual gross premium in the first year and 4% in subsequent years.
- $A_{45} = 0.15$
- $p_{45} = 0.99$
- $i = 0.04$

Let G be the annual gross premium

Calculate the annual gross premium for this policy.

(a) 0.67

(b) 0.70

(c) 0.73

(d) 0.77

(e) 0.80

$$APV(FP_0) = APV(FB_0) + APV(FE_0)$$

$$G\,\ddot{a}_{46} = 100 A_{46} + .06 G + .04 G\,\ddot{a}_{46}$$

$$G(.96\,\ddot{a}_{46} - .06) = 100 A_{46}$$

$$G = 100 A_{46} / (.96\,\ddot{a}_{46} - .06)$$

now apply recursion to derive A_{46}

$$A_{45} = v q_{45} + v p_{45} A_{46} \Rightarrow A_{46} = \frac{A_{45} - v q_{45}}{v p_{45}}$$

$$= \frac{0.15 - (1/1.04)(1-.99)}{(1/1.04)(.99)}$$

$$= 0.1474747$$

and use relationship

$$\ddot{a}_{46} = \frac{1 - A_{46}}{d} = \frac{1 - 0.1474747}{.04/1.04} = 22.16566$$

Finally, plug values

$$G = \frac{100(0.1474747)}{(.96(22.16566) - .06)} = 0.6950117 \approx \underline{\underline{0.70}}$$

Question No. 3: (10 points)

For a special fully discrete whole life insurance issued to (50), you are given:

- The death benefit is $1,000 plus the return of all premiums paid without interest.
- $i = 0.05$
- $(IA)_{50} = 9.268$
- Based on the Equivalence Principle, the level annual premium for this insurance is equal to $ 38.491.

Calculate $\ddot{a}_{50}$.

(a) 6.6

(b) 8.6

(c) 11.2

(d) 13.8

(e) 15.8

$APV(FP_0) = APV(FB_0)$ — return of premium

$P\ddot{a}_{50} = 1000 \underbrace{A_{50}}_{1-d\ddot{a}_{50}} + P(IA)_{50}$

Rearranging, we get

$\ddot{a}_{50}(P + 1000d) = 1000 + P(IA)_{50}$

$\ddot{a}_{50} = \dfrac{1000 + P(IA)_{50}}{P + 1000d}$

$= \dfrac{1000 + (38.491)(9.268)}{38.491 + 1000(.05/1.05)}$

$= 15.75582 \approx \underline{\underline{15.8}}$

Question No. 4: (10 points)

For a special type of whole life insurance issued to (30), you are given:

- Death benefits are 5,000 for the first 10 years and 1,000 thereafter.
- Death benefits are payable at the moment of death.
- Deaths are uniformly distributed over each year of age interval.
- $i = 5\%$
- The following table of actuarial present values:

x	$1000A_x$	$1000\,{}_5E_x$
30	112.31	779.79
35	138.72	779.20
40	171.93	777.14

Calculate the Actuarial Present Value (APV) of the benefits for this policy.

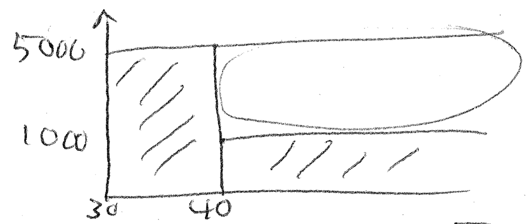
(a) 25.90

(b) 147.25

(c) 399.28

(d) 438.08

(e) 468.42



$$\text{APV(benefits)} = 5000\,\bar{A}_{30} - 4000\,{}_{10}E_{30}\,\bar{A}_{40}$$

$$= 5000\,\frac{i}{\delta}A_{30} - 4000\,{}_{10}E_{30}\,\frac{i}{\delta}A_{40}$$

$$= 5000\left(\frac{.05}{\log(1.05)}\right)(.11231) - 4000(.77979)(.77920) * \left(\frac{.05}{\log(1.05)}\right)(.17193)$$

$$= 147.2457 \approx \underline{\underline{147.25}}$$

Question No. 5: (10 points)

You are given:

- $p_x = 0.99$
- $p_{x+1} = 0.98$
- $p_{x+2} = 0.96$
- ${}_4p_x = 0.89$
- ${}_3p_{x+1} = 0.92$

Calculate ${}_2p_{x+1}$.

(a) 0.960

(b) 0.963

(c) 0.966

(d) 0.969

(e) 0.972

$$\underbrace{{}_3p_{x+1}}_{\text{given}} = {}_2p_{x+1} \cdot \underbrace{p_{x+3}}_{\text{need this}}$$

$$p_x \, p_{x+1} \, p_{x+2} \, p_{x+3} = {}_4p_x$$

$$p_{x+3} = \frac{{}_4p_x}{p_x \, p_{x+1} \, p_{x+2}} = \frac{0.89}{.99(.98)(.96)}$$

$$= 0.955559$$

$${}_2p_{x+1} = \frac{{}_3p_{x+1}}{p_{x+3}}$$

$$= \frac{0.92}{0.955559}$$

$$= 0.9627872 \approx \underline{0.963}$$

DEFECTIVE

Question No. 6: (10 points)

You are given:

- $A_{x+20} = 0.40$
- ${}_{20}E_x = 0.50$
- $A_{x:\overline{20}|} = 0.55$
- $i = 0.03$

Calculate A_x.

(a) 0.05

(b) 0.15

(c) 0.25

(d) 0.40

(e) 0.50

$$A_x = A^1_{x:\overline{20}|} + {}_{20}E_x\, A_{x+20}$$

$$= A_{x:\overline{20}|} - {}_{20}E_x + {}_{20}E_x\, A_{x+20}$$

$$= A_{x:\overline{20}|} - {}_{20}E_x\,(1 - A_{x+20})$$

$$= 0.55 - 0.50(1 - 0.40)$$

$$= \underline{\underline{0.25}}$$

Question No. 7: (10 points)

Get-a-Life Insurance Company sells 10,000 fully discrete whole life insurance policies of \$1, each with the same age 50. You are given:

- All policies have independent future lifetime.
- $A_{50} = 0.300$
- ${}^2A_{50} = 0.125$
- $i = 0.05$
- Premium is determined according to the portfolio percentile principle, with the probability that the total future loss on the portfolio is negative is at least 95%.
- The 95th percentile of a standard Normal distribution is 1.645.

Calculate the annual premium for each policy.

(a) 0.0204

(b) 0.0207

(c) 0.0210

(d) 0.0213

(e) 0.0216

$$L_{agg} = \sum_{i=1}^{10{,}000} L_{0,i} \quad \text{where } L_{0,i} = v^{K+1} - P\ddot{a}_{\overline{K+1}|}$$

(per policy)

$$= v^{K+1}\left(1+\frac{P}{d}\right) - \frac{P}{d}$$

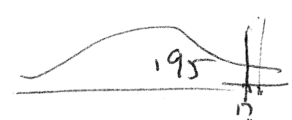
$$E[L_{0,i}] = A_{50} - P\ddot{a}_{50} = 0.300 - P\underbrace{\left(\frac{1-.300}{.05/1.05}\right)}_{14.7}$$

$$= 0.3 - 14.7P$$

$$Var[L_{0,i}] = \left(1+\frac{P}{d}\right)^2\left({}^2A_{50} - A_{50}^2\right)$$

$$= \left(1+P/d\right)^2\underbrace{\left(.125 - 0.3^2\right)}_{0.035}$$

$$E[L_{agg}] = 10{,}000\,(.3-14.7P)$$



$$Var[L_{agg}] = 10{,}000\left(1+P/d\right)^2(.035)$$

$$\therefore \Pr[L_{agg} < 0] \geq 0.95 \Rightarrow \Pr\Bigg[Z < \underbrace{\frac{0-10{,}000(.3-14.7P)}{\sqrt{10{,}000\left(1+P/d\right)^2(.035)}}}_{\geq 1.645}\Bigg] \geq 0.95$$

Solving for P, with $d = \frac{.05}{1.05}$ $\Rightarrow$ $-10{,}000\,(.3-14.7P) \geq 1.645(100)\left(1+\frac{P}{d}\right)\sqrt{.035}$

$$\Rightarrow P\left(147000 - \frac{1.645(100)\sqrt{.035}}{d}\right) \geq 1.645(100)\sqrt{.035} + 3000 \Rightarrow P \geq \underline{0.0207088}$$

Question No. 8: (10 points)

Consider a life (x) with curtate future lifetime denoted by K. A fully discrete whole life insurance is issued to (x) where:

- The death benefit is \$100.
- Expenses, to be paid at the beginning of each year, consist of 4% of each premium.
- The annual premium is G.
- Denote the discount rate by $d = \dfrac{i}{1+i}$.

Which of the following is the loss-at-issue random variable?

(a) $\left(100 + \dfrac{1.04G}{d}\right) v^{K+1} - \dfrac{1.04G}{d}$

(b) $\left(100 - \dfrac{1.04G}{d}\right) v^{K+1} + \dfrac{1.04G}{d}$

(c) $100\, v^{K+1} - 1.04\, G\, \ddot{a}_{\overline{K+1}|}$

(d) $\left(100 - \dfrac{0.96G}{d}\right) v^{K+1} + \dfrac{0.96G}{d}$

(e) $\left(100 + \dfrac{0.96G}{d}\right) v^{K+1} - \dfrac{0.96G}{d}$

$$L_0 = PVFB_0 + PVFE_0 - PVFP_0$$

$$= 100 v^{K+1} + .04\, G\, \ddot{a}_{\overline{K+1}|} - G\, \ddot{a}_{\overline{K+1}|}$$

$$= 100 v^{K+1} - .96 G\, \underbrace{\ddot{a}_{\overline{K+1}|}}_{\frac{1-v^{K+1}}{d}}$$

$$= \left(100 + \frac{.96G}{d}\right) v^{K+1} - \frac{.96G}{d}$$

Question No. 9: (10 points)

You are given:

- $\ddot{a}_x = 3.65$
- $\ddot{a}_{x+1} = 3.55$
- $p_x = 0.80$

Calculate i.

(a) 2%

(b) 4%

(c) 7%

(d) 15%

(e) 20%

Use recursion formula:

$$\ddot{a}_x = 1 + v p_x \ddot{a}_{x+1}$$

$$\frac{\ddot{a}_x - 1}{p_x \ddot{a}_{x+1}} = v \Rightarrow i = \frac{p_x \ddot{a}_{x+1}}{\ddot{a}_x - 1} - 1$$

$$= \frac{.8(3.55)}{3.65 - 1} - 1$$

$$= .0716981$$

7%

Question No. 10: (10 points)

A fully discrete whole life policy of \$100 is issued to (50). Level annual premium is determined with the following expense assumptions:

	% of Premium	Per 100	Per Policy
First year	20%	0.12	2.0
Renewal years	5%	0.07	1.0

Mortality follows the *Illustrative Life Table* with interest rate $i = 6\%$.

Calculate the gross annual premium for this policy.

(a) 2.0

(b) 2.6

(c) 3.2

(d) 3.8

(e) 4.4

$APV(FP_0) = APV(FB_0) + APV(FE_0)$

$G\ddot{a}_{50} = 100A_{50} + .15G + .05G\ddot{a}_{50} + .05 + .07\ddot{a}_{50} + 1 + \ddot{a}_{50}$

Rearranging, we get

$G(.95\ddot{a}_{50} - .15) = 100A_{50} + 1.05 + 1.07\ddot{a}_{50}$

$$G = \frac{100A_{50} + 1.05 + 1.07\ddot{a}_{50}}{.95\ddot{a}_{50} - 0.15}$$

$$= \frac{100(.24905) + 1.05 + 1.07(13.2668)}{.95(13.2668) - 0.15}$$

$$= 3.224042$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK