MATH 3630

Actuarial Mathematics I

Class Test 2 - Section 1/2

Wednesday, 14 November 2012, 8:30-9:30 PM

Time Allowed: 1 hour

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: SUGGESTED SOLUTIONS

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

You are given:

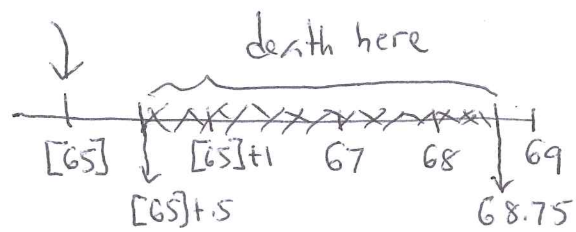
- An extract of a select and ultimate life table with a 2-year select period:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
65	80,625	79,954	78,330	67
66	79,137	78,402	77,252	68
67	77.575	76,778	75,578	69

- During the select period, deaths follow a constant force of mortality over each year of age.

- After the select period, deaths are uniformly distributed over each year of age.

Calculate ${}_{0.5|3.25}q_{[65]}$ and interpret this probability.



$$= \frac{l_{[65]+.5} - l_{68.75}}{l_{[65]}}$$

$$= \frac{l_{[65]}^{.5}\, l_{[65]+1}^{.5} - (.25\, l_{68} + .75\, l_{69})}{l_{[65]}}$$

$$= \frac{(80625)^{.5}(79954)^{.5} - (.25(77252) + .75(75578))}{80625}$$

$$= \frac{80288.8 - 75996.5}{80625} = \underline{\underline{0.0532378 2}}$$

This gives the probability that a select person at 65 will die between the ages of 65.5 and 68.75.

Question No. 2:

Suppose you are given:

- $q_{50} = 0.0025$ $\Rightarrow$ $p_{50} = .9975$
- $q_{51} = 0.0030$ $\Rightarrow$ $p_{51} = .9970$
- $e_{51.6} = 29.1$
- Deaths are uniformly distributed over each year of age.

Calculate $e_{50.6}$. Use the recursive formula for e_x: $e_x = p_x(1+e_{x+1})$

so that $e_{50.6} = p_{50.6}(1+e_{51.6}) = p_{50.6}(1+29.1)$

$= p_{50.6}(30.1)$

where

$$p_{50.6} = \frac{\ell_{51.6}}{\ell_{50.6}} = \frac{(.6\ell_{52}+.4\ell_{51})/\ell_{50}}{(.6\ell_{51}+.4\ell_{50})/\ell_{50}}$$

$$= \frac{.6\,p_{50}\,p_{51} + .4\,p_{50}}{.6\,p_{50}+.4} = \frac{.6(.9975)(.9970)+.4(.9975)}{.6(.9975)+.4}$$

$$= 0.9972003$$

alternatively, $p_{50.6} = {}_{.4}p_{50.6} * {}_{.6}p_{51}$ $\rightarrow$ $p_{50} = {}_{.6}p_{50} * {}_{.4}p_{50.6}$

$$= \frac{p_{50}}{{}_{.6}p_{50}} * {}_{.6}p_{51}$$

$$= p_{50} * \frac{(1-.6\,q_{51})}{(1-.6\,q_{50})} = .9975 * \frac{(1-.6(.0030))}{(1-.6(.0025))}$$

0.9972003 same answer

Finally, $e_{50.6} = .9972003(30.1) = \underline{\underline{30.01573}}$

Question No. 3:

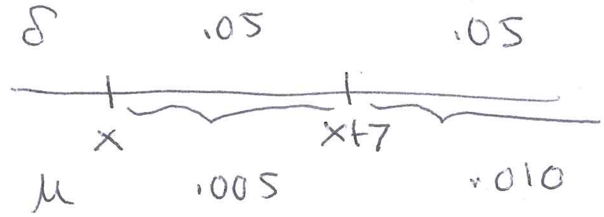
For a whole life insurance of $100 on (x) with benefits payable at the moment of death, you are given:

$$\delta_t = 0.05, \text{ for all } t > 0$$

and

$$\mu_{x+t} = \begin{cases} 0.005, & 0 < t \leq 7 \\ 0.010, & t > 7 \end{cases}$$

Calculate the actuarial present value for this insurance.



$$\text{APV(insurance)} = 100* \underbrace{\frac{.005}{.055}\left(1-e^{-.055(7)}\right)}_{0.02904994} + \underbrace{e^{-.055(7)}\left(\frac{.010}{.060}\right)}_{0.1134084}$$

$$= 100* \underline{\underline{0.1424584}}$$

multiply this by \$100

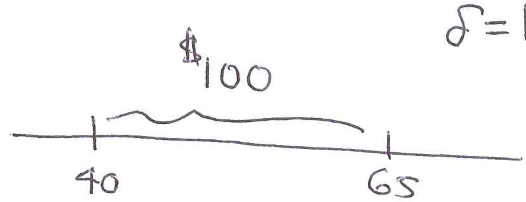
$$= \underline{\underline{14.24584}}$$

Question No. 4:

Mr. Ow Sum is currently age 40. His mortality follows De Moivre's law with $\omega = 110$. He buys a temporary life insurance policy that pays him a benefit of \$100 at the moment of his death, if he dies within the next 25 years. No benefits are made if death occurs after 25 years. You are given that $i = 3.5\%$.

Calculate the actuarial present value of his death benefit.

$\delta = \log(1.035)$



$T_{40} \sim$ Uniform on $(0, 70)$

$$f_T(t) = {}_tp_{40}\,\mu_{40+t} = \frac{1}{70}, \quad 0 \le t \le 70$$

$$\text{APV(death benefit)} = 100 \int_0^{25} e^{-[\log(1.035)]t} \frac{1}{70}\, dt$$

$$= 100 * \frac{1}{70} * \frac{1}{\log(1.035)} \left[1 - e^{-[\log(1.035)](25)}\right]$$

$$= \underline{\underline{23.9547}}$$

Question No. 5:

You are given:

- Z is the present value random variable for a 30-year pure endowment of \$100 issued to (35).
- Mortality follows the Illustrative Life Table.
- $i = 5\%$

Calculate Var[Z].

$$Z = v^{30}\,\underbrace{I(T>30)}_{\text{Bernoulli with probability of success } {}_{30}p_{35}}$$

$$\mathrm{Var}[Z] = (100)^2 * (v^{30})^2 \, {}_{30}p_{35}(1 - {}_{30}p_{35})$$

$$= (100)^2 * \left(\frac{1}{1.05}\right)^{60} \frac{l_{65}}{l_{35}}\left(1 - \frac{l_{65}}{l_{35}}\right)$$

$$= (100)^2 * \left(\frac{1}{1.05}\right)^{60} \frac{7533964}{9420657}\left(1 - \frac{7533964}{9420657}\right)$$

$$= (100)^2 * .008574414$$

$$= \underline{\underline{85.74414}}$$

Question No. 6:

A club consists of n members all age x today. The club has unanimously agreed that starting today:

- A pooled fund will be established to pay a death benefit of \$100 at the end of the year of death of each member.
- Each member will contribute a one-time amount of \$50 to this pooled fund.

You are given the following values: $A_x = 0.455$ and ${}^2A_x = 0.235$. Assume that no member will leave the club prior to death.

Using Normal approximation, determine the smallest n so that there is at least a 0.95 probability that the pooled fund will be sufficient to cover the present value of all promised death benefits.

Total pooled fund = $50n$

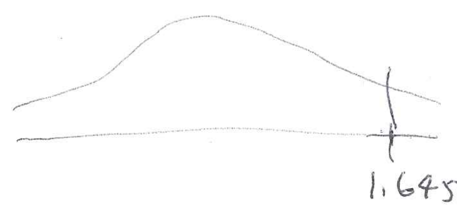
Total present value = $\underbrace{\sum_{i=1}^{n} 100 Z_i}_{\text{Let this be } L}$ where Z_i = p.v. of a whole life of \$1 payable at e.o.y. death

$$E[L] = 100 \cdot n \cdot A_x = 100 \cdot n \cdot (0.455) = 45.5n$$

$$Var[L] = 100^2 \cdot n\left[{}^2A_x - (A_x)^2\right]$$
$$= 100^2 n\left[.235 - (.455)^2\right] = 279.75n$$

$$\underbrace{Pr[L \le 50n]}\geq 0.95$$

$$\approx Pr\left[Z \le \frac{\overbrace{50n - 45.5n}^{4.5n}}{\sqrt{279.75n}}\right] \geq 0.95$$



$$\frac{4.5\cancel{n}^{\sqrt{n}}}{\sqrt{279.75\cancel{n}}} \geq 1.645 \iff \sqrt{n} \geq \frac{1.645\sqrt{279.75}}{4.5} = 6.114183$$

$$\iff n \geq 37.38323$$

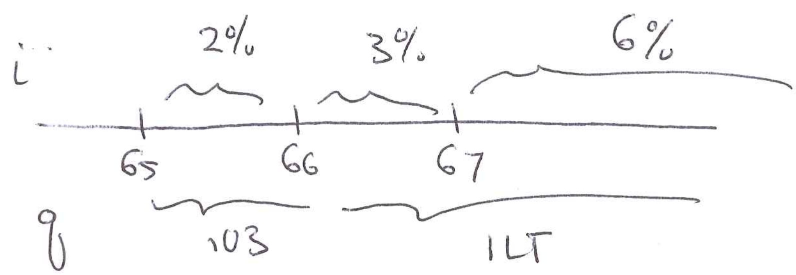
at least 38 members

Question No. 7:

For a whole life insurance of $1,000 issued to (65), you are given:

- Death benefits are payable at the end of the year of death.
- Mortality follows the `Illustrative Life Table` with the exception of the first year where you are given that $q_{65} = 0.03$.
- The annual effective interest rate is 2% in the first year, 3% in the second year, and 6% each year thereafter.

Calculate the actuarial present value of the death benefits.



$$\text{APV(death benefits)} = 1000 * \left[\frac{1}{1.02} q_{65} + \frac{1}{1.02}\frac{1}{1.03} p_{65}\, q_{66}^{ILT} + \frac{1}{1.02}\frac{1}{1.03} p_{65}\, p_{66}^{ILT} A_{67}^{ILT} \right]$$

$$= 1000 * \left[\frac{1}{1.02}(.03) + \frac{1}{1.02}\frac{1}{1.03}(.97)\left(\frac{23.29}{1000}\right) + \frac{1}{1.02}\frac{1}{1.03}(.97)\left(1 - \frac{23.29}{1000}\right)(.46947) \right]$$

$$= 1000 * 0.474273$$

$$= \underline{\underline{474.273}}$$

Question No. 8:

$i^{(12)} = 12(1.05^{1/12} - 1) = .04888949$

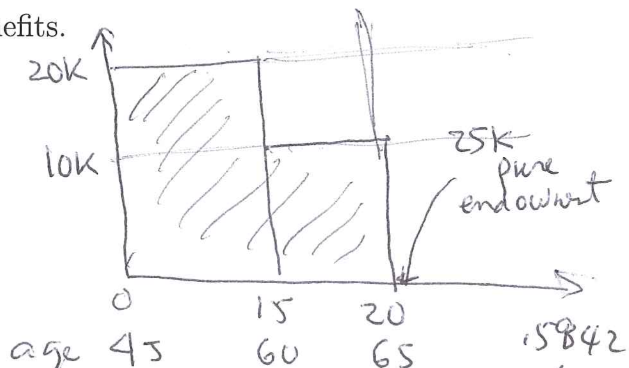
Leo is currently age 45 who purchases a special endowment insurance policy which will pay him:

- \$20,000 at the end of the month of his death, if death occurs during the next 15 years,
- \$10,000 at the end of the month of his death, if death occurs the following 5 years, and
- \$25,000 at the end of 20 years, if alive.

You are given:

- Mortality follows the `Standard Ultimate Survival Model`.
- $i = 5\%$
- Deaths are uniformly distributed between integral ages.

Calculate the actuarial present value of Leo's insurance benefits.



Use Std Ult Surv Model table @ 5%

APV(Leo's benefits)

$$= \underbrace{\frac{i}{i^{(12)}}}_{.05 / .04888949} \Big[20{,}000 \underbrace{A_{45}}_{.15161} - 10{,}000 \, \underbrace{{}_5E_{45}}_{.77991} \, \underbrace{{}_{10}E_{50}}_{.60182} \, \underbrace{A_{60}}_{.29028} - 10{,}000 \, \underbrace{{}_{10}E_{45}}_{.60655} \, \underbrace{{}_{10}E_{55}}_{.59842} \, \underbrace{A_{65}}_{.35477} \Big]$$

$$+ 25{,}000 \, \underbrace{{}_{10}E_{45}}_{.60655} \, \underbrace{{}_{10}E_{55}}_{.59342}$$

$$= \underbrace{401.6925}_{\text{term}} + \underbrace{8998.473}_{\text{pure endowment}} = \underline{\underline{9400.165}}$$

Question No. 9:

For a cohort of individuals all age x consisting of 75% non-smokers (ns) and 25% smokers (sm), you are given:

k	q_{x+k}^{ns}	q_{x+k}^{sm}
0	0.01	0.08
1	0.03	0.12
2	0.05	0.20

Calculate $A_{x:\overline{2|}}^{\;1}$ for a randomly chosen individual from this cohort. You are given: $i = 3\%$.

$$^{ns}A_{x:\overline{2|}}^{1} = v\, q_x^{ns} + v^2\, p_x^{ns}\, q_{x+1}^{ns}$$

$$= \frac{1}{1.03}\, 0.01 + \frac{1}{1.03^2}(.99)(.03) = 0.03770384$$

$$^{sm}A_{x:\overline{2|}}^{1} = v\, q_x^{sm} + v^2\, p_x^{sm}\, q_{x+1}^{sm}$$

$$= \frac{1}{1.03}\, 0.08 + \frac{1}{1.03^2}(.92)(.12) = 0.1817325$$

$$A_{x:\overline{2|}}^{1} = .75(0.03770384) + .25(0.1817325)$$

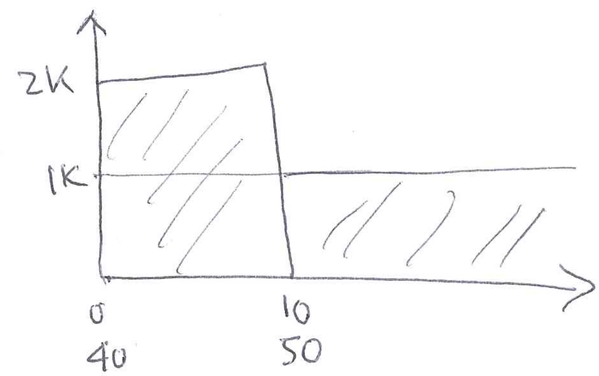
$$= \underline{\underline{0.073711}}$$

Question No. 10:

For a special whole life insurance on (40), you are given:

- Death benefit is payable at the end of the year of death.
- Death benefit is $2,000 during the first 10 years, and $1,000 thereafter.
- Mortality follows the `Illustrative Life Table`.
- $i = 6\%$

Calculate the Actuarial Present Value of this insurance.



$$\text{APV(insurance)} = 2000\, \underbrace{A_{40}}_{.16132} - 1000\, \underbrace{{}_{10}E_{40}}_{.53667}\, \underbrace{A_{50}}_{.24905}$$

$$= \underline{\underline{188.9823}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK