MATH 3630
Actuarial Mathematics I
Class Test 2
Wednesday, 17 November 2010
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: SUGGESTED SOLUTIONS

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

For a life (x), you are given:

- $v = 0.95$
- $q_{x+k} = 0.04$, for all $k = 0, 1, 2, \ldots$

Calculate a_x and interpret this value.

Using current payment technique,

$$a_x = \sum_{k=1}^{\infty} v^k \, {}_kp_x = \sum_{k=1}^{\infty} (.95)^k (.96)^k$$

$$= \sum_{k=1}^{\infty} (.912)^k$$

$$= \frac{.912}{1-.912} = \underline{\underline{10.36364}}$$

This gives the actuarial present value of a life annuity of \$1 each year to (x), with first payment at the end of the first year continuing for life thereafter.

Question No. 2:

You are given:

- the following extract from a life table:

x	95	96	97	98	99	100
ℓ_x	10	7	4	2	1	0

- $i = 5\%$
- Y is the present value random variable of a (discrete) 2-year deferred 2-year temporary life annuity-due of 1 per year to (95)

Calculate the variance of Y.

payments 0 0 1 1 0 →

95 96 97 98 99

$$Y = \begin{cases} v^2, & \text{w.p. } p_{95}\, p_{96}\, q_{97} \longrightarrow \dfrac{l_{97}}{l_{95}}\left(1 - \dfrac{l_{98}}{l_{97}}\right) = .2 \\ v^2+v^3, & \text{w.p. } p_{95}\, p_{96}\, p_{97} \longrightarrow \dfrac{l_{98}}{l_{95}} = \dfrac{2}{10} = .2 \end{cases}$$

Thus,

$$E(Y) = .2v^2 + .2(v^2+v^3) = .4v^2 + .2v^3 = .5355793$$

$$E(Y^2) = .2v^4 + .2(v^2+v^3)^2 = .7917345$$

$$Var(Y) = .7917345 - (.5355793)^2$$

$$= \underline{\underline{.5048893}}$$

Question No. 3:

You and a friend are studying together for the MLC actuarial exam. One of the practice problems asked to compute the value for $\ddot{a}_{50}$ based on $i = 4\%$.

Your friend calculated the value and came up with 13.5. But the correct answer turned out to be 13.3. He asked you to review his work and you discovered that he used the following set of mortality assumptions:

$$q_{50} = 0.02 \text{ and } q_{51} = 0.03.$$

You realized then that all assumptions in his calculations were correct, except for q_{51}.

What must have been the correct value for q_{51}?

Using recursion, the correct calculation should have been

$$\ddot{a}_{50} = 1 + v p_{50} + v^2 p_{50} p_{51} \ddot{a}_{52}$$

$$13.3 = 1 + \frac{1}{1.04}(.98) + \frac{1}{1.04^2}(.98) p_{51} \ddot{a}_{52}$$

$\ddot{a}_{52}$ in both calculations are the same, so using your friend's calculations, you can similarly derive

$$\ddot{a}_{52} = \frac{13.5 - (1 + \frac{1}{1.04}(.98))}{\frac{1}{1.04^2}(.98)(.97)} = 13.15043$$

Solving for p_{51}, we get

$$p_{51} = \frac{13.3 - (1 + \frac{1}{1.04}(.98))}{\frac{1}{1.04^2}(.98)(13.15043)} = .953 2147$$

$\Rightarrow$ correct q_{51} is $q_{51} = 1 - .9532147 = \underline{\underline{.0467853}}$

Question No. 4:

You are given: implies constant force

- Mortality follows an Exponential distribution with $\mu = 0.05$.
- The constant force of interest is $\delta = 3\%$.
- Y is the present value random variable for a continuous whole life annuity on (45).

Calculate the variance of Y.

$$\mathrm{Var}(Y) = \frac{1}{\delta^2}\left({}^2\bar{A}_{45} - \bar{A}_{45}^2\right)$$

where

$$\bar{A}_{45} = \frac{\mu}{\mu+\delta} = \frac{.05}{.08} = 5/8$$

$$^2\bar{A}_{45} = \frac{\mu}{\mu+2\delta} = \frac{.05}{.11} = 5/11$$

$$= \frac{1}{.03^2}\left[\frac{5}{11} - \left(\frac{5}{8}\right)^2\right]$$

$$= \underline{\underline{71.02273}}$$

Question No. 5:

The following two policies are actuarially equivalent:

- a special whole life insurance on (30) payable at the end of the year of death with benefits of

  10,000 if death occurs in the first 10 years,

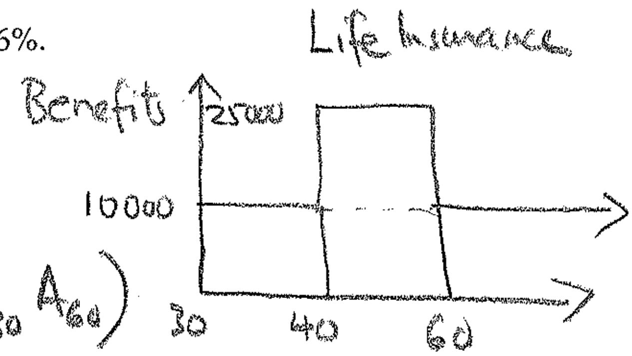
  25,000 if death occurs the following 20 years, and

  back to 10,000 thereafter.

- a special whole life annuity-due on (30) payable once a year where the benefit payments are B in the first 10 years and increasing to $2B$ thereafter.

Mortality follows the *Illustrative Life Table* with $i = 6\%$.

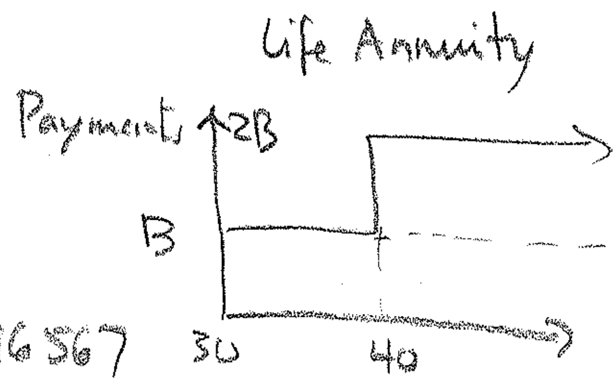
Calculate B.



APV(Life Insurance)

$= 10{,}000 A_{30} + 15{,}000 \left({}_{10}E_{30} A_{40} - {}_{30}E_{30} A_{60} \right)$

$= 10(102.48) + 15(547.33)(.16132 - .27414(.36913)) = 1518.437$



APV(Life Annuity)

$= B \ddot{a}_{30} + B \, {}_{10}E_{30} \ddot{a}_{40}$

$= B\left(\ddot{a}_{30} + {}_{10}E_{30} \ddot{a}_{40} \right) = 23.96567$

$\quad 15.8561 \quad .54733 \quad 14.8166$

The two are equivalent, hence

$B = \dfrac{1518.437}{23.96567} = \underline{\underline{63.35885}}$

Question No. 6:

The t-party club consists of 1,000 members all age x today. The club has unanimously agreed that starting today:

- A pooled fund will be established to pay a death benefit of \$10 at the moment of death of each member.
- Each member will contribute a one-time amount of c to the fund so that there is a 97% probability all promised payments are met when due.

You are given the following values: $\bar{A}_x = 0.20$, ${}^2\bar{A}_x = 0.06$, and $\delta = 5\%$.

Using Normal approximation, determine c.

Let S = aggregate loss $= 10(Z_1 + Z_2 + \ldots + Z_{1000})$

where Z_i = P.V. r.v. of a WLI of 1 on (x) payable at the moment of death

$$E(Z_i) = \bar{A}_x = .2$$

$$Var(Z_i) = {}^2\bar{A}_x - \bar{A}_x^2 = .06 - .2^2 = .06 - .04 = .02$$

Therefore $E(S) = 10 * 1000 * (.2) = 2000$

$$Var(S) = 10^2 * 1000 * (.02) = 2000$$

$$Pr(S \leq 1000c) \cong Pr\left(Z \leq \underbrace{\frac{1000c - 2000}{\sqrt{2000}}}_{1.88}\right) = .97$$

$$\frac{1000c - 2000}{\sqrt{2000}} = 1.88 \Rightarrow c = \frac{2000 + 1.88\sqrt{2000}}{1000} = \underline{\underline{2.084076}}$$

Question No. 7:

For a life (x), you are given:

$$\mu_{x+t} = \begin{cases} 0.04, & 0 < t \leq 10 \\ 0.08, & t > 10 \end{cases}$$

and

$$\delta_t = \begin{cases} 0.03, & 0 < t \leq 10 \\ 0.05, & t > 10 \end{cases}$$

Calculate $\bar{a}_{\overline{x:\overline{10}|}}$ and <u>interpret this value</u>.

$$\bar{a}_{\overline{x:\overline{10}|}} = \bar{a}_{\overline{10}|} + {}_{10|}\bar{a}_x$$

$$= \bar{a}_{\overline{10}|} + {}_{10}E_x \, \bar{a}_{x+10}$$

$$= \int_0^{10} e^{-.03t} \, dt + e^{-.03(10)} e^{-.04(10)} \int_0^{\infty} e^{-.05t} e^{-.08t} \, dt$$

$$= \frac{1}{.03}\left(1 - e^{-.3}\right) + e^{-.7} \cdot \frac{1}{.13}$$

$$= 12.45928$$

This gives the APV of a continuous whole life annuity of 1 per year on (x) with the first 10 years of guaranteed payments.

Question No. 8:

You are given:

- Z is the present value random variable for a whole life insurance of 1 payable at the moment of death of (50).
- Mortality follows de Moivre's law.
- $\delta = 5\%$
- The probability that Z exceeds 0.0734 is 0.95.

Calculate ω in the de Moivre's law.

Let T_{50} be the future lifetime of (50). Then

$T_{50} \sim$ Uniform on $(0, \omega - 50)$.

Thus, $\Pr(Z > .0734) = \Pr\left(e^{-\delta T_{50}} > .0734\right)$

$$= \underbrace{\Pr\left(T_{50} < \frac{-1}{.05}\log(.0734)\right)}_{\dfrac{\frac{-1}{.05}\log(.0734)}{\omega - 50}} = .95$$

$$\frac{\frac{-1}{.05}\log(.0734)}{\omega - 50} = .95$$

$$\Rightarrow \quad \omega = \frac{-1}{.05(.95)}\log(.0734) + 50$$

$$= 104.9859 \approx \underline{\underline{105}}$$

Question No. 9:

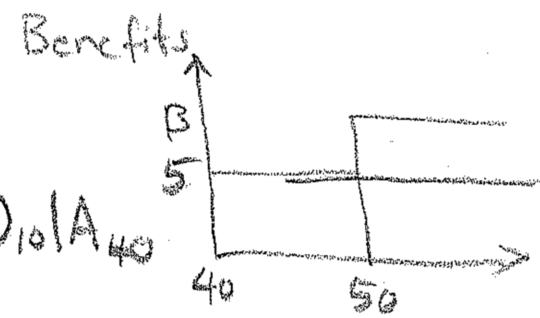
Two life insurance policies to be issued to (40) are actuarially equivalent:

- A whole life insurance of 10 payable at the end of the year of death.
- A special whole life insurance, also payable at the end of the year of death, that pays 5 for the first 10 years and B thereafter.

You are given:

- $i = 4\%$
- $10A_{40} = 3.0$
- $10A_{50} = 3.5$
- $10A^{\,1}_{40:\overline{10}|} = 0.9$

Calculate the value of B.



$$\text{APV(first policy)} = 10A_{40} = 3$$

$$\text{APV(second policy)} = 5A_{40} + (B-5)\,{}_{10|}A_{40}$$

$$= \frac{3}{2} + (B-5)\,{}_{10}E_{40}\,\underbrace{A_{50}}_{.35}$$

Solving for B, we get

$$B = 5 + \frac{3/2}{.35\,{}_{10}E_{40}} = 5 + \frac{3/2}{.35(.6)} = \underline{\underline{12.14286}}$$

$$A^{\,1}_{40:\overline{10}|} = A_{40} - {}_{10}E_{40}\,A_{50}$$

$$\Rightarrow {}_{10}E_{40} = \frac{A_{40} - A^{\,1}_{40:\overline{10}|}}{A_{50}} = \frac{.3 - .09}{.35} = .6$$

Question No. 10:

Maria will retire today at exactly age 60. She has accumulated a total savings of \$500,000. Using all her total savings, she purchased a policy which will pay her:

- a monthly life annuity benefit of b for life with the first payment to start at the end of the first month, plus
- a death benefit of \$200,000 payable at the moment of her death.

You are given:

- Mortality follows the *Illustrative Life Table* with $i = 6\%$.
- Deaths are assumed to be uniformly distributed over each year of age.

Calculate b.

The APV of her policy is

$$12b\, a_{60}^{(12)} + 200000\, \bar{A}_{60} = \underbrace{12b\, \ddot{a}_{60}^{(12)} - b + 200000\bar{A}_{60}}_{\text{should equal to her total savings of } 500000}$$

Solving for b, we get

$$b = \frac{500000 - 200000\, \bar{A}_{60}}{12\ddot{a}_{60}^{(12)} - 1} \qquad i = 6\%, \quad \delta = \log(1.06)$$

$$= \frac{500000 - 200000 \frac{i}{\delta} \overset{.36913}{A_{60}}}{12(\underset{1.00028}{\alpha(12)}\, \underset{11.1454}{\ddot{a}_{60}} - \underset{0.46812}{\beta(12)}) - 1} \quad \text{from UDD}$$

$$= \underline{\underline{3,334.10}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK