MATH 3630
Actuarial Mathematics I
Class Test 2
Wednesday, 18 November 2009
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: ________________________ **Student ID:** _________________

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

For a whole life insurance of a benefit of 100 on (30) payable at the moment of death, you are given:

$$\mu_{30+t} = 0.01, \quad \text{for all } t > 0$$

and

$$\delta_t = \begin{cases} 0.04, & \text{for } 0 < t \leq 5 \\ 0.05, & \text{for } t > 5 \end{cases} .$$

Calculate the Actuarial Present Value of the benefit for this life insurance.

Question No. 2:

Let Z_1 denote the present value random variable of an n-year term insurance of \$1, while Z_2 that of an n-year deferred insurance of \$1, with death benefit payable at the moment of death of (x).

You are given:

$$\bar{A}^{\,1}_{x:\overline{n|}} = 0.01 \qquad {}^2\bar{A}^{\,1}_{x:\overline{n|}} = 0.0005$$

and

$${}_{n|}\bar{A}_x = 0.10 \qquad {}^2_{n|}\bar{A}_x = 0.0136$$

Calculate the correlation coefficient of Z_1 and Z_2.

In case you forgot: $\mathrm{Corr}(Z_1, Z_2) = \dfrac{\mathrm{Cov}(Z_1, Z_2)}{\sqrt{\mathrm{Var}(Z_1)\mathrm{Var}(Z_2)}}$.

Question No. 3:

For a special life insurance on (30), you are given:

- mortality follows the *Illustrative Life Table* with $i = 6\%$;
- death benefit is payable at the end of the year of death; and
- death benefit is 100 for the first 10 years, and 300 thereafter.

Calculate the Actuarial Present Value of this insurance.

Question No. 4:

An insurance company has a portfolio of 100 whole life insurance policies all issued to age x. You are given:

- The future lifetimes of the policyholders are independent.
- The benefit of each policyholder is \$5 payable at the end of the year of death.
- $A_x = 0.35$
- ${}^2A_x = 0.15$
- $i = 4\%$

Using Normal approximation, calculate the amount of the fund needed at issue in order to be 95% certain of having enough money to pay the death benefits. (Note: the 95$^{\text{th}}$ percentile of a standard Normal is 1.645.)

Question No. 5:

Suppose you are given:

$$_kp_x = (0.95)^k, \text{ for } k = 0, 1, \ldots$$

Let Y denote the present value random variable of a whole life annuity-due of \$1 each year issued to (x). Calculate the expected value of Y if $i = 10\%$.

Question No. 6:

For a group of insured individuals all age x, let Z be the present value random variable of a whole life insurance that pays $1 at the end of the year of death. The group is made up of half male and half female.

You are given:

$$A_x^{\mathrm{M}} = 0.16 \qquad A_x^{\mathrm{F}} = 0.08$$

and

$$\mathrm{Var}(Z|\mathrm{M}) = 0.04 \qquad \mathrm{Var}(Z|\mathrm{F}) = 0.05$$

where 'M' denotes male and 'F' denotes female.

Calculate the (unconditional) variance $\mathrm{Var}(Z)$ for an individual randomly selected from the group.

Question No. 7:

For a whole life annuity-due of $100 on (x), payable annually, you are given:

- $q_x = 0.01$
- $q_{x+1} = 0.05$
- $i = 0.05$
- $\ddot{a}_{x+1} = 6.951$

Calculate the change in the Actuarial Present Value of this annuity-due if p_{x+1} is increased by 0.02. Did the Actuarial Present Value increase of decrease? Use your intuition to explain why.

Question No. 8:

You are given:

- T_x has an exponential distribution with mean 20.
- ${}^2\bar{a}_x = 5.75$

Calculate the force of interest δ.

Question No. 9:

For a special 20-year temporary life annuity-due issued to (55), you are given:

- Mortality follows the *Illustrative Life Table* with $i = 6\%$.
- The benefits are \$100 each year for the first 10 years, and \$500 each year the last 10 years.

Calculate the Actuarial Present Value of the benefits for this life annuity.

Question No. 10:

Raul is currently age 40. His mortality follows DeMoivre's law with $\omega = 100$. He purchases a whole life insurance policy that pays a benefit of \$1 at the moment of death.

Compute the variance of the present value of his death benefit if $\delta = 5\%$.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK