Michigan State University

STT 455 - Actuarial Models I

Class Test 1

Monday, 20 October 2014

Total Marks: 100 points

Please write your name at the space provided:

Name: EMIL VALDEZ

- There are five (5) multiple choice (MC) and one (1) written-answer questions here and you are to answer all questions asked. Points assigned are clearly indicated on each question.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

MC Question No. 1: (10 points)

Your actuarial colleague is using the following survival model to price insurance products:

$$S_0(x) = \frac{120}{120 - x}, \quad \text{for } 0 < x < 120,$$

where x is measured in years.

You realize that this is not a legitimate survival function. As you proceed to your colleague's office to explain this, you were thinking about the arguments you could make that this model is inappropriate.

Which of the following statements would be your "strongest" argument to make your case?

(a) A newborn is not alive with certainty at birth.

(b) The average age at death for a newborn is only 20 years.

(c) The probability of survival increases with age. ✓

(d) The probability of survival decreases with age.

(e) No one has really survived beyond 115 years old.

$$\frac{d}{dx} S_0(x) = \frac{120}{(120-x)^2} > 0$$

increasing with age

MC Question No. 2: (10 points)

You are given:

- The probability that (x) survives another 10 years is 0.75.
- The probability that (x) survives another 15 years is 0.25.

For a person who has reached age $x + 10$, calculate the probability that he will die within the following 5 years.

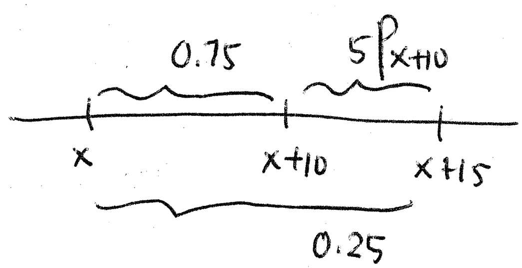
(a) 1/4

(b) 1/3

(c) 2/3 ✓

(d) 3/4

(e) cannot be determined from the given information



Using multiplicative property of p's, we get

$$0.75 \; {}_5p_{x+10} = 0.25$$

$$ {}_5p_{x+10} = \frac{0.25}{0.75} = \frac{1}{3}$$

So that $\; {}_5q_{x+10} = 1 - \frac{1}{3} = \cancel{3/4} \; 2/3$

MC Question No. 3: (10 points)

Suppose you are given the following select-and-ultimate mortality table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	x+3
20	35,600	35,591	35,583	35,572	23
21	35,587	35,576	35,562	35,553	24
22	35,574	35,561	35,547	35,534	25

Calculate the probability that a life with select age 21 will survive for one year but die the following two years.

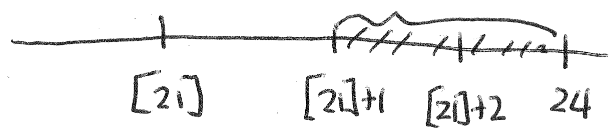
(a) 0.0000

(b) 0.0002

(c) 0.0004

(d) 0.0006 ✓

(e) 0.0008



Required probability is

$$\frac{\ell_{[21]+1} - \ell_{24}}{\ell_{[21]}} = \frac{35576 - 35553}{35587}$$

$$= \underline{.000646303 4}$$

MC Question No. 4: (10 points)

Suppose you are given the following extract from a life table:

x	ℓ_x
52	89948
53	89089
54	88176
55	87208
56	86181

Deaths are uniformly distributed over each year of age interval.

Calculate the probability that a person now age 52.5 will survive another 2.75 years.

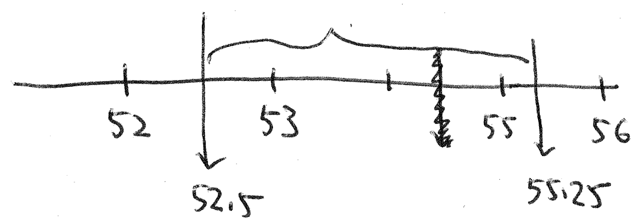
(a) 0.9656

(b) 0.9685

(c) 0.9713 ✓

(d) 0.9737

(e) 0.9752



$${}_{2.75}p_{52.5} = \frac{l_{55.25}}{l_{52.5}} = \frac{.25\, l_{56} + .75\, l_{55}}{.5(l_{52}+l_{53})}$$

$$= \frac{.25(86181) + .75(87208)}{.5(89948 + 89089)}$$

$$= \underline{0.9713216}$$

Question No. 5: (10 points)

A life insurance policy issued to (x) provides the following benefits:

- Payments are made at the moment of death.
- If death occurs within the first 10 years, the benefit amount is 10.
- If (x) survives the following 10 years, a pure endowment of 20 will be provided at the end of 10 years.
- There is no benefit paid for deaths after 10 years.

Let T be the future lifetime of (x) and v is the usual one-year discount function.

Which of the following gives the present value random variable associated with this policy?

(a) $Z = \begin{cases} 20v^T, & T < 10 \\ 0, & T \geq 10 \end{cases}$

(b) $Z = \begin{cases} 10v^T, & T < 10 \\ 20v^T, & T \geq 10 \end{cases}$

(c) $Z = \begin{cases} 10v^T, & T < 10 \\ 20v^{10}, & T \geq 10 \end{cases}$ ✓

(d) $Z = \begin{cases} 10v^T, & T < 10 \\ 30v^{10}, & T \geq 10 \end{cases}$

(e) $Z = \begin{cases} 20v^{10}, & T < 10 \\ 10v^T, & T \geq 10 \end{cases}$

this is the term part

Clearly $\begin{cases} 10v^T, & T<10 \\ 20v^{10}, & T\geq 10 \end{cases}$

this is the pure endowment part

There are four (4) parts to the written-answer portion of this test. You are to answer all parts. Please provide as much details of your calculations as possible to get your partial points for any incorrect answers.

The survival function of a newborn is given by

$$S_0(x) = 1 - \frac{x}{110}, \quad \text{for } 0 \le x \le 110.$$

(i) (10 points) Find the force of mortality at age x, μ_x.

$$\mu_x = -\frac{1}{S_0(x)} \frac{d}{dx} S_0(x)$$

$$= \frac{\cancel{-}1}{1-\frac{x}{110}} \cdot \frac{\cancel{-}1}{110} = \frac{1/110}{\frac{110-x}{110}} = \frac{1}{110-x}, \quad 0 \le x \le 110$$

(ii) (7 points) Find an expression for ${}_tp_{40}$.

$${}_tp_{40} = S_{40}(t) = \frac{S_0(40+t)}{S_0(40)}$$

$$= \frac{1-\frac{40+t}{110}}{1-\frac{40}{110}} = \frac{70-t}{70} = 1-\frac{t}{70},$$

$$0 \leq t \leq 70$$

(iii) (8 points) Find an expression for the density function $f_{40}(t)$. Are you surprised with this expression? Why or why not?

$$f_{40}(t) = \frac{-d}{dt} S_{40}(t) = -\left(\frac{1}{-70}\right) = \frac{1}{70}, \quad 0 \leq t \leq 70$$

OR

$$ {}_t p_{40}\, \mu_{40+t} = \left(1 - t/70\right)\left(\frac{1}{110-40-t}\right) = \frac{\cancel{70-t}}{70} \frac{1}{\cancel{70-t}}$$

$$= \frac{1}{70}$$

Same answer

Not surprising result. This tells us that T_{40} is uniform on $[0, 70]$.

$\Rightarrow$ de Moivre's

From $S_0(x) = 1 - \frac{x}{110} = \frac{110-x}{110}$,

we could get $f_0(x) = \frac{1}{110}$

which is de Moivre's to start with!

(iv) (25 points) Suppose $\delta = 5\%$. A whole life insurance policy to (40) provides a benefit of 100 at his moment of death. Calculate the actuarial present value for this insurance. Give a final numerical value. Use the result in part (iii).

$$Z = 100e^{-.05T}$$

$$APV = E(Z) = \int_0^{70} 100e^{-.05t} \cdot \frac{1}{70}\,dt$$

$$= \frac{100}{70}\left(\frac{-1}{.05}\right)\left(e^{-.05t}\Big|_0^{70}\right)$$

$$= \frac{100}{70}\left(\frac{-1}{.05}\right)\left(e^{-.05(70)} - 1\right)$$

$$= \underline{\underline{27.70865}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK