

Michigan State University
STT 455 - Actuarial Models I
Class Test 1
Monday, 7 October 2013
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____ Section No.: _____

- There are five (5) multiple choice (MC) and one (1) written-answer questions here and you are to answer all questions asked. Points assigned are clearly indicated on each question.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

MC Question No. 1: (10 points)

You are given the survival function of a newborn:

$$S_0(x) = 1 - \left(\frac{x}{100}\right)^2, \quad \text{for } 0 \leq x \leq 100.$$

Calculate the probability that a life (30) will die within the next 10 years.

- (a) 0.08
- (b) 0.09
- (c) 0.12
- (d) 0.15
- (e) 0.16

MC Question No. 2: (10 points)

You are given:

$$\mu_x = \begin{cases} 0.01, & 0 < x < 20 \\ 0.02, & x \geq 20 \end{cases}$$

Calculate ${}_8p_{15}$.

- (a) 0.852
- (b) 0.878
- (c) 0.896
- (d) 0.914
- (e) 0.923

MC Question No. 3: (10 points)

In a population consisting of 45% males and 55% females, you are given:

- Mortality for males has a constant force of μ .
- Mortality for females also has a constant force of 0.80μ .
- The probability a female survives one year is 0.98.

Calculate the percentage of the surviving population who are females at the end of 20 years.

- (a) 0.55
- (b) 0.56
- (c) 0.57
- (d) 0.58
- (e) 0.59

MC Question No. 4: (10 points)

You are given the following extract from a select and ultimate life table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x + 2$
44	30,053	29,873	29,601	46
45	29,615	29,417	29,131	47
46	29,130	28,919	28,614	48
47	28,600	28,377	28,052	49

Assume that deaths are uniformly distributed between integral ages.

Calculate $1000_{0.6}q_{[45]+0.7}$.

- (a) 4.5
- (b) 4.6
- (c) 4.7
- (d) 4.8
- (e) 4.9

Question No. 5: (10 points)

In a two-year select and ultimate mortality table, you are given:

- $q_{[x]+1} = 0.96 q_{x+1}$
- $l_{50} = 985$
- $l_{51} = 900$

Calculate $l_{[49]+1}$.

- (a) 900
- (b) 904
- (c) 940
- (d) 981
- (e) 985

There are five (5) parts to the written-answer portion of this test. All parts are related to life tables and you are to answer all parts. Please provide as much details of your calculations as possible to get your partial points for any incorrect answers.

(i) (10 points) Please fill the rest of the life table below:

x	l_x	d_x	p_x	q_x
79	100			
80	85			
81	65			
82	35			
83	0	na	na	na

na = not applicable

(ii) (10 points) Using the life table in part (i) and assuming uniform distribution of deaths between ages 81 and 82 and constant force of mortality between ages 82 and 83:

- (a) Calculate $q_{81.5}$.
- (b) Explain in words what this value means.

- (iii) (10 points) Let K_{80} be the curtate future lifetime of an 80-year-old. Using the life table in part (i), calculate the probability distribution of K_{80} by filling the empty spaces in the table below:

k	$\Pr[K_{80} = k]$
0	
1	
2	

(iv) (10 points) Using the results in part (iii), calculate the expected value of K_{80} .

(v) (10 points) Using the life table in part (i), calculate e_{80} using the formula

$$e_{80} = \sum_{k=1}^{\infty} k p_{80}$$

and show that it matches exactly the result you have in part (iv).

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK