Michigan State University

STT 455 - Actuarial Models I

Class Test 1

Monday, 7 October 2013

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTIONS Section No.: ________________

- There are five (5) multiple choice (MC) and one (1) written-answer questions here and you are to answer all questions asked. Points assigned are clearly indicated on each question.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

MC Question No. 1: (10 points)

You are given the survival function of a newborn:

$$S_0(x) = 1 - \left(\frac{x}{100}\right)^2, \quad \text{for } 0 \le x \le 100.$$

Calculate the probability that a life (30) will die within the next 10 years.

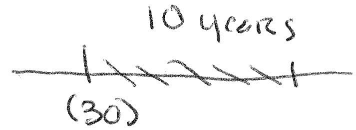
✓ (a) 0.08

(b) 0.09

(c) 0.12

(d) 0.15

(e) 0.16



$$_{10}p_{30} = \frac{S_0(40)}{S_0(30)}$$

$$= \frac{1-(.4)^2}{1-(.3)^2}$$

$$= \frac{.84}{.91} = 0.9230769$$

$$_{10}q_{30} = 1 - {}_{10}p_{30}$$

$$= 1 - 0.9230769$$

$$= .07692308 \approx \underline{\underline{.08}}$$

MC Question No. 2: (10 points)

You are given:

$$\mu_x = \begin{cases} 0.01, & 0 < x < 20 \\ 0.02, & x \geq 20 \end{cases}$$

Calculate ${}_8p_{15}$.

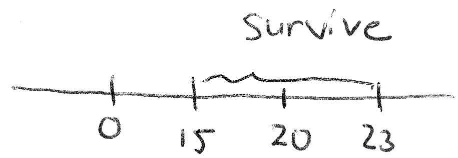
(a) 0.852

(b) 0.878

✓(c) 0.896

(d) 0.914

(e) 0.923



$$ {}_8p_{15} = {}_5p_{15} \cdot {}_3p_{20} $$

$$ = e^{-\int_0^5 .01 ds} \; e^{-\int_0^3 .02 ds} $$

$$ = e^{-.01(5)} \; e^{-.02(3)} $$

$$ = e^{-.11} $$

$$ = 0.8958341 \approx \underline{\underline{0.896}} $$

MC Question No. 3: (10 points)

In a population consisting of 45% males and 55% females, you are given:

- Mortality for males has a constant force of μ.
- Mortality for females also has a constant force of 0.80μ.
- The probability a female survives one year is 0.98. $\Rightarrow \quad e^{-\mu} = .98$

Calculate the percentage of the surviving population who are females at the end of 20 years.

(a) 0.55

(b) 0.56

(c) 0.57

(d) 0.58

(e) 0.59

Probability male live to 20 is $e^{-20\mu}$

probability female live to 20 is $e^{-.80(20\mu)} = e^{-16\mu}$

$$\text{Proportion of female} = \frac{0.55\, e^{-16\mu}}{0.55\, e^{-16\mu} + 0.45\, e^{-20\mu}}$$

$$= \frac{.55\,(.98)^{16}}{.55(.98)^{16} + .45(.98)^{20}}$$

$$= 0.569094 \approx \underline{\underline{0.57}}$$

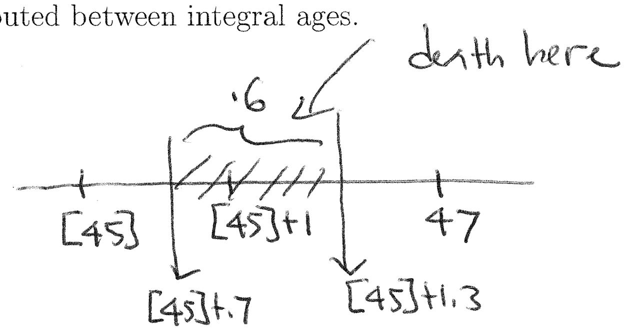
MC Question No. 4: (10 points)

You are given the following extract from a select and ultimate life table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
44	30,053	29,873	29,601	46
45	29,615	29,417	29,131	47
46	29,130	28,919	28,614	48
47	28,600	28,377	28,052	49

Assume that deaths are uniformly distributed between integral ages.

Calculate $1000\,{}_{0.6}q_{[45]+0.7}$.



(a) 4.5

(b) 4.6

(c) 4.7

(d) 4.8

✓(e) 4.9

$$1000\,{}_{0.6}q_{[45]+.7} = 1000\left[1 - {}_{.6}p_{[45]+.7}\right]$$

$$= 1000\left[1 - \frac{\ell_{[45]+1.3}}{\ell_{[45]+.7}}\right]$$

$$= 1000\left[1 - \frac{.3\,\ell_{47} + .7\,\ell_{[45]+1}}{.7\,\ell_{[45]+1} + .3\,\ell_{[45]}}\right]$$

$$= 1000\left[1 - \frac{.3(29131) + .7(29417)}{.7(29417) + .3(29615)}\right]$$

$$= \underline{\underline{4.925975}}$$

Question No. 5: (10 points)

In a two-year select and ultimate mortality table, you are given:

- $q_{[x]+1} = 0.96\, q_{x+1}$ $\Longrightarrow p_{[x]+1} = 1 - 0.96(1 - p_{x+1})$
- $\ell_{50} = 985$
- $\ell_{51} = 900$

Set $x = 49$

Calculate $\ell_{[49]+1}$.

$$\frac{\ell_{51}}{\ell_{[49]+1}} = 1 - 0.96\left(1 - \frac{\ell_{51}}{\ell_{50}}\right)$$

(a) 900

(b) 904

(c) 940

✓(d) 981

(e) 985

$$\frac{900}{\ell_{[49]+1}} = \underbrace{1 - 0.96\left(1 - \frac{900}{985}\right)}_{0.9171574}$$

$$\ell_{[49]+1} = \frac{900}{0.9171574}$$

$$= \underline{\underline{981.2929}}$$

There are five (5) parts to the written-answer portion of this test. All parts are related to life tables and you are to answer all parts. Please provide as much details of your calculations as possible to get your partial points for any incorrect answers.

(i) (10 points) Please fill the rest of the life table below:

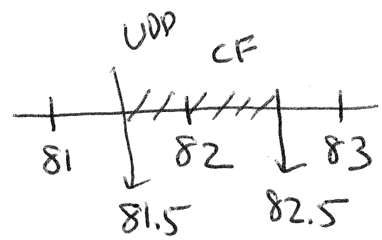
x	ℓ_x	d_x	p_x	q_x
79	100	15	.850	$\frac{15}{100}$ = .150
80	85	20	.765	$\frac{20}{85}$ = .235
81	65	30	.538	$\frac{30}{65}$ = .462
82	35	35	0	$\frac{35}{35}$ = 1
83	0	na	na	na

na = not applicable

(ii) (10 points) Using the life table in part (i) and assuming uniform distribution of deaths between ages 81 and 82 and constant force of mortality between ages 82 and 83:

(a) Calculate $q_{81.5}$.

(b) Explain in words what this value means.



$$q_{81.5} = 1 - p_{81.5} = 1 - \frac{l_{82.5}}{l_{81.5}}$$

$$= 1 - \frac{l_{82}^{.5}\, l_{83}^{.5}}{.5(l_{81} + l_{82})}$$

$$= 1 - \frac{35^{.5}\, 0^{.5}}{.5(65+35)} = 1$$

This gives the probability that a person now age 81.5 will not $\boxed{\text{survive}}$ to reach 82.5! or die before reaching 82.5!

(iii) (10 points) Let K_{80} be the curtate future lifetime of an 80-year-old. Using the life table in part (i), calculate the probability distribution of K_{80} by filling the empty spaces in the table below:

k	$\Pr[K_{80} = k]$
0	$\frac{d_{80}}{l_{80}} = \frac{20}{85} = .235$
1	$\frac{d_{81}}{l_{80}} = \frac{30}{85} = .353$
2	$\frac{d_{82}}{l_{80}} = \frac{35}{85} = .412$

One can show that $\Pr[K_{80}=k] = {}_kp_{80}\, q_{80+k}$

$$= \frac{l_{80+k}}{l_{80}} \left[\frac{l_{80+k} - l_{80+k+1}}{l_{80+k}} \right]$$

$$= \frac{d_{80+k}}{l_{80}}$$

(iv) (10 points) Using the results in part (iii), calculate the expected value of K_{80}.

$$E[K_{80}] = \cancel{0(.235)} + 1(.353) + 2(.412)$$

$$= \underline{\underline{1.177}}$$

(v) (10 points) Using the life table in part (i), calculate e_{80} using the formula

$$e_{80} = \sum_{k=1}^{\infty} {}_kp_{80}$$

and show that it matches exactly the result you have in part (iv).

$$e_{80} = p_{80} + {}_2p_{80} + {}_3p_{80}$$

$$= \frac{l_{81}}{l_{80}} + \frac{l_{82}}{l_{80}} + \frac{\cancel{l_{83}} \to 0}{l_{80}}$$

$$= \frac{65}{85} + \frac{35}{85}$$

$$= \frac{100}{85}$$

$$= \underline{\underline{1.176471}} \approx 1.177$$ which matches part (iv)

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK