MATH 3630
Actuarial Mathematics I
Class Test 1 - Section 2 - 3:00-4:15 PM
Wednesday, 3 October 2012
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____   Student ID: _____

- There are ten (10) written-answer questions here and you are to answer all
  ten. Each question is worth 10 points.

- Please provide details of your workings in the appropriate spaces provided;
  partial points will be granted.

- Please write legibly.

- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

For a fixed age x, you are given the following probabilities:

- $_{20}p_x = 0.40$

- $_5p_x = 0.78$

- $_5p_{x+15} = 0.75$

Calculate $_{15}q_{x+5}$.

Question No. 2:

Suppose that the survival function for a newborn is given by

$$S_0(x) = \left[(1+x)\,\mathrm{e}^{-x}\right]^{1/100}, \quad \text{for } x \geq 0.$$

Calculate μ_{50}.

Question No. 3:

In a population consisting of 75% non-smokers and 25% smokers, you are given:

- Mortality for non-smokers has a constant force of μ.

- Mortality for smokers also has a constant force of 2μ, twice that of non-smokers.

- The probability a non-smoker survives a year is 0.96.

What proportion of the surviving population are smokers at the end of 10 years?

Question No. 4:

The force of mortality for a substandard life (x) is expressed as

$$\mu^s_{x+t} = \mu_{x+t} + c,$$

for some constant $c > 0$, where μ_{x+t} is the force of mortality of a standard life (x).

You are given:

- The probability that a standard life (x) survives the next 5 years is 0.75.

- The probability that a substandard life (x) survives the next 5 years is 0.40.

Calculate the value of c.

Question No. 5:

Mortality follows Generalized De Moivre's law expressed as:

$$S_0(x) = \left(1 - \frac{x}{110}\right)^{1/4}, \text{ for } 0 \leq x \leq 110.$$

Calculate $_{10|10}q_{40}$ and <u>interpret this probability</u>.

Question No. 6:

You are given the following probabilities for a life (50):

t	$_tp_{50}$
0.5	0.985
1.0	0.960
1.5	0.948
2.0	0.932

Using repeated Simpson's rule with $n = 2$ intervals, estimate the value of $\mathring{e}_{50:\overline{2}|}$.

Question No. 7:

For a population which consists of 75% non-smokers (NS) and 25% smokers (SM) at age 30, you are given:

t	$S_{30}^{\text{NS}}(t)$	$S_{30}^{\text{SM}}(t)$
20	0.40	0.30
21	0.38	0.25

Calculate p_{50} for a randomly chosen individual from this population.

Question No. 8:

You are given:

$$q_{50+k} = 0.02, \quad \text{for } k = 0, 1, 2, \ldots$$

Calculate $E[K_{50}]$, where K_{50} is the curtate future lifetime of (50).

Question No. 9:

You are given:

$$\mu_x = \begin{cases} 0.02, & 0 < x < 10 \\ 0.05, & x \geq 10 \end{cases}$$

Calculate $_{10}p_5$.

Question No. 10:

You are given:

- Mortality follows De Moivre's law.

- $\mathring{e}_{50} = 27.5$

Calculate $\text{Var}[X]$.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK