MATH 3630
Actuarial Mathematics I
Class Test 1 - Section 2 - 3:00-4:15 PM
Wednesday, 3 October 2012
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL   Student ID: SUGGESTED SOLUTIONS

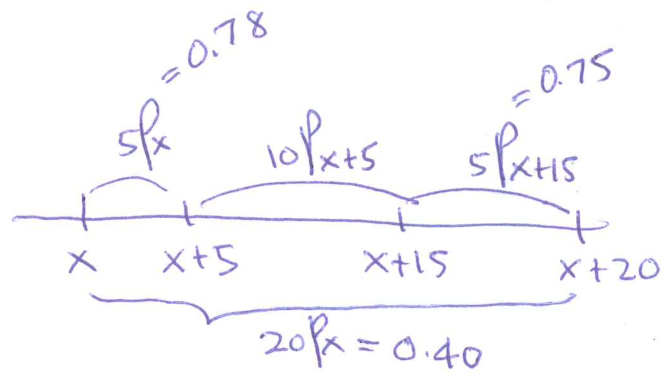
- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

For a fixed age x, you are given the following probabilities:

- ${}_{20}p_x = 0.40$
- ${}_{5}p_x = 0.78$
- ${}_{5}p_{x+15} = 0.75$

Calculate ${}_{15}q_{x+5}$.



$${}_{20}p_x = {}_5p_x * {}_{10}p_{x+5} * {}_5p_{x+15}$$

$${}_{10}p_{x+5} = \frac{{}_{20}p_x}{{}_5p_x * {}_5p_{x+15}} = \frac{0.40}{0.78(0.75)}$$

$${}_{15}q_{x+5} = 1 - {}_{15}p_{x+5} = 1 - {}_{10}p_{x+5} * {}_5p_{x+15}$$

$$= 1 - \frac{0.40}{0.78(\cancel{0.75})}(\cancel{0.75}) = 1 - \frac{.40}{.78}$$

$$= \frac{.38}{.78} = \frac{38}{78} = \frac{19}{39} = \underline{\underline{0.4871795}}$$

Question No. 2:

Suppose that the survival function for a newborn is given by

$$S_0(x) = \left[(1+x)\,\mathrm{e}^{-x}\right]^{1/100}, \quad \text{for } x \geq 0.$$

Calculate μ_{50}.

$$\log S_0(x) = \frac{1}{100}\left[\log(1+x) + \log(e^{-x})\right]$$

$$= \frac{1}{100}\left[\log(1+x) - x\right]$$

$$\mu_x = -\frac{d}{dx}\log S_0(x) = -\frac{1}{100}\left[\frac{1}{1+x} - 1\right]$$

$$= -\frac{1}{100}\left[\frac{-x}{1+x}\right] = \frac{1}{100}\left(\frac{x}{1+x}\right)$$

$$\mu_{50} = \frac{1}{\cancel{100}_{\,2}}\left(\frac{\cancel{50}}{51}\right) = \frac{1}{102} = \underline{\underline{0.009803922}}$$

Question No. 3:

In a population consisting of 75% non-smokers and 25% smokers, you are given:

- Mortality for non-smokers has a constant force of μ.
- Mortality for smokers also has a constant force of 2μ, twice that of non-smokers.
- The probability a non-smoker survives a year is 0.96.

What proportion of the surviving population are smokers at the end of 10 years?

$\Rightarrow \quad e^{-\mu} = 0.96$

at the end of 10 years, the proportion of smokers is

$$\frac{\overset{1/4}{0.25}\, e^{-20\mu}}{\underset{1/4}{0.25}\, e^{-20\mu} + \underset{3/4}{0.75}\, e^{-10\mu}} = \frac{e^{-10\mu}}{e^{-10\mu}+3} = \frac{(0.96)^{10}}{(0.96)^{10}+3} = \underline{0.1814087}$$

Note that because of constant force, it does not matter what the ages are of the individuals in the population.

Question No. 4:

The force of mortality for a substandard life (x) is expressed as

$$\mu^s_{x+t} = \mu_{x+t} + c,$$

for some constant $c > 0$, where μ_{x+t} is the force of mortality of a standard life (x). You are given:

- The probability that a standard life (x) survives the next 5 years is 0.75.
- The probability that a substandard life (x) survives the next 5 years is 0.40.

Calculate the value of c.

Standard life: $\quad {}_5p_x = e^{-\int_0^5 \mu_{x+t}\,dt} = 0.75$

substandard life: $\quad {}_5p^s_x = e^{-\int_0^5 (\mu_{x+t}+c)\,dt}$

$$= e^{-\int_0^5 \mu_{x+t}\,dt}\ e^{-5c}$$

$$= {}_5p_x\, e^{-5c} = 0.75\, e^{-5c} = 0.40$$

Solving for c, we get

$$e^{5c} = \frac{.75}{.40} = \frac{15}{8} \Rightarrow 5c = \log\left(\frac{15}{8}\right)$$

$$c = \frac{1}{5}\log\left(\frac{15}{8}\right)$$

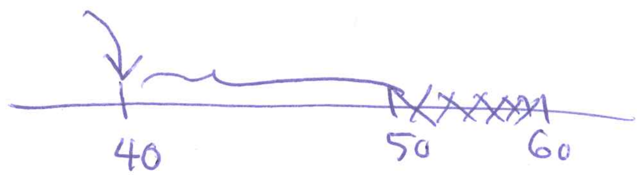
$$= \underline{\underline{0.1257217}}$$

Question No. 5:

Mortality follows Generalized De Moivre's law expressed as:

$$S_0(x) = \left(1 - \frac{x}{110}\right)^{1/4}, \text{ for } 0 \leq x \leq 110.$$

Calculate ${}_{10|10}q_{40}$ and <u>interpret this probability</u>.



$${}_{10|10}q_{40} = {}_{10}p_{40} - {}_{20}p_{40}$$

$$= \frac{S_0(50) - S_0(60)}{S_0(40)} = \frac{\left(\frac{60}{\cancel{110}}\right)^{1/4} - \left(\frac{50}{\cancel{110}}\right)^{1/4}}{\left(\frac{70}{\cancel{110}}\right)^{1/4}}$$

$$= \frac{60^{1/4} - 50^{1/4}}{70^{1/4}} = \underline{\underline{0.0428727 4}}$$

this gives the probability that (40) will survive the next 10 years but die the following 10 years, or (40) dies between the ages of 50 and 60.

Question No. 6:

You are given the following probabilities for a life (50):

t	${}_tp_{50}$
0.5	0.985
1.0	0.960
1.5	0.948
2.0	0.932

Using repeated Simpson's rule with $n = 2$ intervals, estimate the value of $\mathring{e}_{50:\overline{2}|}$.

$$\mathring{e}_{50:\overline{2}|} = \int_0^2 {}_tp_{50}\,dt = \int_0^1 {}_tp_{50}\,dt + \int_1^2 {}_tp_{50}\,dt$$

$$\approx \frac{1/2}{3}\left[{}_0p_{50} + 4\,{}_{0.5}p_{50} + p_{50}\right]$$

$$+ \frac{1/2}{3}\left[p_{50} + 4\,{}_{1.5}p_{50} + {}_2p_{50}\right]$$

$$= \frac{1}{6}\left[1 + 4\,{}_{0.5}p_{50} + 2p_{50} + 4\,{}_{1.5}p_{50} + {}_2p_{50}\right]$$

$$= \frac{1}{6}\left[1 + 4(0.985) + 2(0.960) + 4(0.948) + 0.932\right]$$

$$= \frac{11.584}{6} = \underline{\underline{1.930667}}$$

Question No. 7:

For a population which consists of 75% non-smokers (NS) and 25% smokers (SM) at age 30, you are given:

t	$S_{30}^{NS}(t)$	$S_{30}^{SM}(t)$
20	0.40	0.30
21	0.38	0.25

Calculate p_{50} for a randomly chosen individual from this population.

$$p_{50} = \frac{S_0(51)}{S_0(50)} = \frac{S_0(51)/S_0(30)}{S_0(50)/S_0(30)} = \frac{S_{30}(21)}{S_{30}(20)}$$

where

$$S_{30}(21) = 0.75(0.38) + 0.25(0.25) = 0.3475$$

$$S_{30}(20) = 0.75(0.40) + 0.25(0.30) = 0.3750$$

Thus,

$$p_{50} = \frac{0.3475}{0.3750} = \underline{\underline{0.9266667}}$$

Question No. 8:

You are given:

$$q_{50+k} = 0.02, \quad \text{for } k = 0, 1, 2, \ldots$$

Calculate $\mathrm{E}[K_{50}]$, where K_{50} is the curtate future lifetime of (50).

$$\mathrm{E}[K_{50}] = e_{50} = \sum_{k=1}^{\infty} {}_kp_{50}$$

$$= \sum_{k=1}^{\infty} \underbrace{p_{50} \cdot p_{51} \cdots\cdots p_{50+k-1}}_{(.98)^k}$$

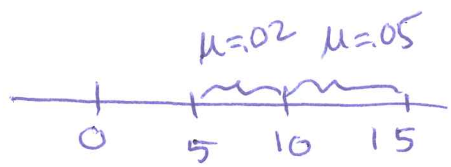
$$= \frac{.98}{1-.98} = \frac{.98}{.02} = \frac{98}{2} = 49$$

Question No. 9:

You are given:

$$\mu_x = \begin{cases} 0.02, & 0 < x < 10 \\ 0.05, & x \geq 10 \end{cases}$$

Calculate ${}_{10}p_5$.



$${}_{10}p_5 = {}_5p_5 * {}_5p_{10}$$

$$= e^{-.02(5)} * e^{-.05(5)}$$

$$= e^{-.35}$$

$$= \underline{\underline{0.7046881}}$$

Question No. 10:

You are given:

- Mortality follows De Moivre's law.
- $\mathring{e}_{50} = 27.5$

Calculate Var[X].

$X \sim$ Uniform on $(0, \omega)$ $\quad E[X] = \omega/2$

$Var[X] = \omega^2/12$

$T_{50} \sim$ Uniform on $(0, \omega - 50)$

$$E[T_{50}] = \frac{\omega - 50}{2} = 27.5 \Rightarrow \omega = 50 + 2(27.5) = 105$$

$$Var[X] = 105^2/12 = \underline{\underline{918.75}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK