MATH 3630
Actuarial Mathematics I
Class Test 1
Wednesday, 5 October 2011
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: SOLUTIONS

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

For a standard life, the force of mortality is

$$\mu_x = \frac{1}{2(100-x)}, \quad \text{for } 0 \le x < 100.$$

For a substandard life, the force of mortality is twice that of a standard life of the same age.

Calculate the probability that a substandard life currently age 50 will survive to reach age 65.

For substandard, $\mu_x^S = 2\mu_x = \frac{1}{100-x}$, $0 \le x < 100$

This implies $T_0^S \sim$ Uniform on $[0,100)$ so that

$T_{50}^S \sim$ Uniform also on $[0,50)$

$$\Pr[T_{50}^S > 15] = 1 - \Pr[T_{50}^S \le 15]$$

$$= 1 - \frac{15}{50} = \frac{35}{50} = 0.70$$

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alternatively, recognize μ_x for standard has a Generalized de Moivre's form with $\alpha = 1/2$, $\omega = 100$

so that $_tp_x = \left(1 - \frac{t}{100-x}\right)^{1/2}$

$$_{15}p_{50} = \Pr[T_{50} > 15] = \left(1 - \frac{15}{50}\right)^{1/2} = (35/50)^{1/2}$$

$$_{15}p_{50}^S = \Pr[T_{50}^S > 15] = \left(_{15}p_{50}\right)^2 = 35/50 = 0.70$$

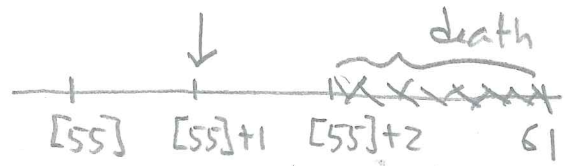
Same answer!

Question No. 2:

Suppose you are given the following select-and-ultimate mortality table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	x+3
55	882	877	871	864	58
56	875	870	863	856	59
57	868	863	856	849	60
58	861	855	848	840	61
59	854	847	840	832	62
60	846	839	832	823	63

Calculate the probability that a life now age 56, with select age 55, will die between the ages of 57 and 61.



$$_{1|4}q_{[55]+1} = \frac{\ell_{[55]+2} - \ell_{61}}{\ell_{[55]+1}}$$

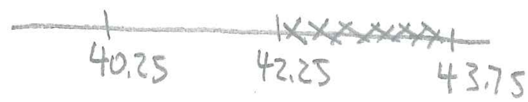
$$= \frac{871 - 840}{877} = \underline{\underline{0.03535}}$$

Question No. 3:

Assume mortality follows the *Illustrative Life Table.*

Suppose that the constant force assumption holds between integral ages.

Calculate ${}_{2|1.5}q_{40.25}$ and interpret this probability.



$$
{}_{2|1.5}q_{40.25} = \frac{l_{42.25} - l_{43.75}}{l_{40.25}}
$$

$$
= \frac{l_{42}^{.75}\, l_{43}^{.25} - l_{43}^{.25}\, l_{44}^{.75}}{l_{40}^{.75}\, l_{41}^{.25}}
$$

you may also directly substitute from here

$$
= \left(\frac{l_{42}}{l_{40}}\right)^{.75}\left(\frac{l_{43}}{l_{41}}\right)^{.25} - \left(\frac{l_{43}}{l_{41}}\right)^{.25}\left(\frac{l_{44}}{l_{40}}\right)^{.75}
$$

$$
= \left(\frac{l_{43}}{l_{41}}\right)^{.25}\left[\left(\frac{l_{42}}{l_{40}}\right)^{.75} - \left(\frac{l_{44}}{l_{40}}\right)^{.75}\right]
$$

$$
= \left(\frac{9229925}{9287264}\right)^{.25}\left[\left(\frac{9259571}{9287264}\right)^{.75} - \left(\frac{9198149}{9313166}\right)^{.75}\right]
$$

$$
= \underline{\underline{0.00495}}
$$

Question No. 4:

You are given:

- $\mathring{e}_{30} = 51.50, \quad \mathring{e}_{35} = 46.68, \quad \text{and} \quad \mathring{e}_{40} = 41.91$
- $\mathring{e}_{30:\overline{5}|} = 4.988 \quad \text{and} \quad \mathring{e}_{30:\overline{10}|} = 9.963$

Calculate ${}_5p_{35}$.

$$\mathring{e}_{30} = \mathring{e}_{30:\overline{5}|} + {}_5p_{30}\,\mathring{e}_{35} \Rightarrow {}_5p_{30} = \frac{\mathring{e}_{30} - \mathring{e}_{30:\overline{5}|}}{\mathring{e}_{35}}$$

$$= \frac{51.50 - 4.988}{46.68}$$

$$= 0.996401$$

$$\mathring{e}_{30} = \mathring{e}_{30:\overline{10}|} + \underbrace{{}_{10}p_{30}}_{{}_5p_{30}\ {}_5p_{35}}\mathring{e}_{40}$$

$$\Rightarrow {}_5p_{35} = \frac{\mathring{e}_{30} - \mathring{e}_{30:\overline{10}|}}{{}_5p_{30}\,\mathring{e}_{40}} = \frac{51.50 - 9.963}{0.996401\,(41.91)}$$

$$= \underline{\underline{0.9946798}}$$

Question No. 5:

You are given:

$$\mu_x = \begin{cases} 0.04, & 0 < x < 10 \\ 0.08, & x \geq 10 \end{cases} \Rightarrow \mu_{5+t} = \begin{cases} .04, & 0<t<5 \\ .08, & t \geq 5 \end{cases}$$

Calculate ${}_{10}p_5$.

$$ {}_{10}p_5 = {}_5p_5 \; {}_5p_{10} $$

$$ = e^{-.04(5)} \cdot e^{-.08(5)} $$

$$ = \underline{\underline{0.5488116}} $$

Question No. 6:

For a life (x), you are given $\ell_x = 1000$ and the following extract from a mortality table:

k	d_{x+k}
0	400
1	200
2	200
3	200

Calculate the variance of K_x.

Recall $Pr[K_x = k] = {}_{k|}q_x = {}_k p_x \, q_{x+k}$

$= \frac{\cancel{l_{x+k}}}{l_x} \frac{d_{x+k}}{\cancel{l_{x+k}}}$

$= d_{x+k} / l_x$

k	$Pr[K_x = k] = \frac{d_{x+k}}{l_x}$
0	.4
1	.2
2	.2
3	.2

$E[K_x] = .2(1+2+3) = 1.2$

$E[K_x^2] = .2(1^2 + 2^2 + 3^2) = 2.8$

$Var[K_x] = 2.8 - 1.2^2 = \underline{\underline{1.36}}$

Question No. 7:

For a population which consists of 60% males (m) and 40% females (f) at birth, you are given:

x	$S_0^{\mathrm{m}}(x)$	$S_0^{\mathrm{f}}(x)$
50	0.08	0.10
51	0.07	0.09

Calculate q_{50} for a randomly chosen individual from this population.

$$q_{50} = 1 - p_{50} = 1 - \frac{S_0(51)}{S_0(50)}$$

$$S_0(50) = .6(.08) + .4(.10) = .088$$

$$S_0(51) = .6(.07) + .4(.09) = .078$$

$$q_{50} = 1 - \frac{.078}{.088} = \underline{\underline{0.1136364}}$$

alternatively, at age 50, determine % male and % female

$$\%\text{male @ } 50 = .60(.08)/[.60(.08) + .40(.10)] = .5454545$$

$$\%\text{female @ } 50 = 1 - .5454545 = 0.4545455$$

$$p_{50} = .5454545\left(\frac{0.07}{0.08}\right) + .4545455\left(\frac{0.09}{0.10}\right) = 0.8863636$$

$$q_{50} = 1 - p_{50} = \underline{\underline{0.1136364}}$$ Same answer!

Question No. 8:

Suppose you are given:

$$S_0(x) = 1 - \left(\frac{x}{100}\right)^2, \quad \text{for } 0 \le x \le 100.$$

Calculate μ_{50}.

$$\mu_x = \frac{-1}{S_0(x)} \frac{d}{dx} S_0(x) = \frac{-1}{1-\left(\frac{x}{100}\right)^2} \cdot \left(\frac{-2x}{100}\right)\frac{1}{100} = \frac{x/5000}{1-\left(\frac{x}{100}\right)^2}$$

$$\mu_{50} = \frac{50/5000}{1-\left(50/100\right)^2} = \frac{1/100}{1-1/4} = \frac{1}{75} = \underline{\underline{0.0133333}}$$

Question No. 9:

You are given the following extract from a select-and-ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

Calculate the probability that a life with select age 62 will survive the next three years.

$${}_3p_{[62]} = p_{[62]}\, p_{[62]+1}\, p_{[62]+2}$$

$$= (1-.11)(1-.13)(1-.15)$$

$$= \underline{\underline{0.658155}}$$

Question No. 10:

You are given:

- $\ell_x = 800$
- $\ell_{x+0.5} = 700$
- $\ell_{x+1} = 600$

Using trapezium (trapezoidal) rule with $h = 0.5$, estimate the one-year temporary expectation of life of (x).

$$\mathring{e}_{x:\overline{1}|} = \int_0^1 {}_tp_x\,dt = \int_0^{.5} {}_tp_x\,dt + \int_{.5}^1 {}_tp_x\,dt$$

$$\approx \frac{1}{4}\left[1 + {}_{.5}p_x\right] + \frac{1}{4}\left[{}_{.5}p_x + {}_1p_x\right]$$

trapezium rule with $h = 1/2$

$$= \frac{1}{4} + \frac{1}{2}\frac{l_{x+.5}}{l_x} + \frac{1}{4}\frac{l_{x+1}}{l_x}$$

$$= \frac{1}{4} + \frac{\frac{1}{2}(700) + \frac{1}{4}(600)}{800}$$

$$= \underline{\underline{0.875}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK