MATH 3630
Actuarial Mathematics I
Class Test 1
Wednesday, 29 September 2010
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: SOLUTIONS

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

A class of species follows a mortality pattern described by

$$S_X(x) = \left(\frac{2}{2+x}\right)^2, \text{ for } x \geq 0,$$

where x is measured in years.
Calculate $\mathring{e}_1$ and interpret this value.

$${}_tp_1 = \frac{S_x(1+t)}{S_x(1)} = \frac{4(3+t)^{-2}}{4/9} = 9(3+t)^{-2}, \quad t \geq 0$$

$$\mathring{e}_1 = \int_0^\infty {}_tp_1 \, dt = -9(3+t)^{-1} \Big|_0^\infty$$

$$= 9/3 = 3$$

this gives the average number of years yet to live for a one-year-old species.

Question No. 2:

Assume the Uniform Distribution of Death (UDD) assumption holds between integral ages. You are given:

$$_{0.5}q_{50} = 0.005$$

$$_{0.4}q_{51} = 0.008$$

Calculate the probability that a life (50) will survive the next two years.

$$_{.5}q_{50} = .5\, q_{50} = .005 \Rightarrow q_{50} = .01$$

$$_{.4}q_{51} = .4\, q_{51} = .008 \Rightarrow q_{51} = .02$$

$$_{2}p_{50} = p_{50}\, p_{51} = .99(.98)$$

$$= 0.9702$$

Question No. 3:

A cohort of newborn currently has mortality pattern described by

$$\mu_x = \begin{cases} 0.015, & \text{for } 0 < x \leq 35 \\ 0.04, & \text{for } x > 35 \end{cases}$$

A medical breakthrough will reduce the force of mortality for age beyond 35 by 25%, but will not affect mortality prior to, and including, age 35.

Calculate the percentage improvement in the probability of a 30-year-old reaching to age 65 as a result of this medical breakthrough.

Without medical breakthrough, consider ${}_tp_{30} = e^{-\int_0^t \mu_{30+s}\,ds}$

where $\mu_{30+s} = \begin{cases} .015, & s \leq 5 \\ .04, & s > 5 \end{cases}$

$${}_{35}p_{30} = e^{-\int_0^5 .015\,ds}\, e^{-\int_5^{35} .04\,ds}$$

$$= e^{-5(.015)-30(.04)} = e^{-1.275} \approx 0.2794$$

With medical breakthrough, $\mu_{30+s} = \begin{cases} .015, & s \leq 5 \\ .03, & s > 5 \end{cases}$

$${}_{35}p_{30} = e^{-5(.015)-30(.03)} = e^{-.975} \approx 0.3772$$

The percentage improvement is therefore

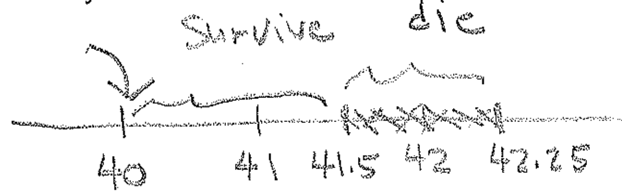
$$\frac{.3772 - .2794}{.2794} = \underline{\underline{35\%}}$$

Question No. 4:

Assume mortality follows the *Illustrative Life Table*.

Suppose that constant force of mortality assumption holds between integral ages.

Calculate ${}_{1.5|0.75}q_{40}$ and interpret this probability.



$$_{1.5|.75}q_{40} = p_{40} * \left[{}_{.5}p_{41} - p_{41} * {}_{.25}p_{42} \right]$$

$$= p_{40} * \left[(p_{41})^{.5} - p_{41} * (p_{42})^{.25} \right]$$

$$= \left(1 - \frac{2.78}{1000}\right)\left[\left(1 - \frac{2.98}{1000}\right)^{.5} - \left(1 - \frac{2.98}{1000}\right)\left(1 - \frac{3.20}{1000}\right)^{.25}\right]$$

$$= .002281704 \approx \underline{.0023}$$

this gives the probability that a 40-year-old will die between the ages of 41.5 and 42.25.

Question No. 5:

For a portfolio of life insurance policies, you are given that *substandard* lives have force of mortality twice that of *standard* lives. You are given that mortality for *standard* lives is described according to

$$\ell_x = 500(110 - x), \text{ for } 0 \le x \le 110.$$

Calculate the probability that a *substandard* life (30) will die within 10 years.

$$\mu_x^s = \frac{-1}{\ell_x}\frac{d}{dx}\ell_x = \frac{-1}{500(110-x)} \cdot (-500) = \frac{1}{110-x}$$

$${}_{10}p_{30}^s = e^{-\int_0^{10} \frac{1}{80-t}\,dt} = e^{\log 70/80} = 7/8$$

Since $\mu_x^{ns} = 2\mu_x^s$, then ${}_{10}p_{30}^{ns} = e^{-2\int_0^{10} \mu_{30+t}^s\,dt}$

$$= \left({}_{10}p_{30}^s\right)^2$$

Thus, ${}_{10}p_{30}^{ns} = \left(\frac{7}{8}\right)^2 = .765625$

the subscripts s, ns refer to standard and substandard, respectively

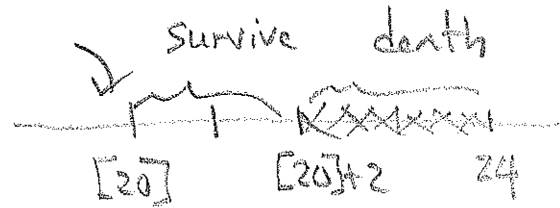
$${}_{10}q_{30}^{ns} = 1 - {}_{10}p_{30}^{ns} = \underline{\underline{.234375}}$$

Question No. 6:

Suppose you are given the following select-and-ultimate mortality table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	x+3
20	35,600	35,591	35,583	35,572	23
21	35,587	35,576	35,562	35,553	24
22	35,574	35,561	35,547	35,534	25

Calculate the probability that a life with select age 20 will survive for two years but die the following two years.



$$_{2|2}q_{[20]} = {}_2p_{[20]} * {}_2q_{[20]+2}$$

$$\frac{\ell_{[20]+2}}{\ell_{[20]}} * \left(1 - \frac{\ell_{24}}{\ell_{[20]+2}}\right)$$

OR $$\frac{\ell_{[20]+2} - \ell_{24}}{\ell_{[20]}} = \frac{35{,}583 - 35{,}553}{35{,}600}$$

$$= .000842696 6$$

$$\approx \underline{\underline{.0008}}$$

Question No. 7:

$\frac{8-x}{8}$

You are given:

$$S_X(x) = 1 - \frac{1}{8}x, \text{ for } 0 \le x \le 8.$$

Calculate ${}_2m_4$.

$$\mu_x = \frac{-1}{S_x(x)} \frac{d}{dx} S_x(x) = \frac{\cancel{-}1}{1-\frac{1}{8}x}\left(\cancel{-}\frac{1}{8}\right) = \frac{1}{8-x}$$

$${}_2m_4 = \frac{\int_4^6 S_x(y)\mu_y\,dy}{\int_4^6 S_x(y)\,dy} = \frac{\int_4^6 \frac{1}{8}\,dy}{\int_4^6 \left(1-\frac{1}{8}y\right)dy}$$

$$= \frac{\frac{1}{\cancel{8}_4}\cancel{(6-4)}^{2}}{2-\frac{1}{\cancel{16}_4}\cancel{(6^2-4^2)}^{\cancel{20}\,5}} = \frac{1/4}{2-5/4} = \frac{1/\cancel{4}}{3/\cancel{4}}$$

$$= \underline{\underline{1/3}}$$

Question No. 8:

$\left(\frac{\omega-x}{\omega}\right)^{1/2}$

The following Generalized de Moivre's law holds:

$$S_X(x) = \left(1 - \frac{x}{\omega}\right)^{1/2}, \quad \text{for } 0 \le x \le \omega.$$

In addition, you are given that the probability a life (25) survives another 10 years is 0.8944.

Calculate ω.

$${}_{10}p_{25} = \frac{S_x(35)}{S_x(25)} = \left(\frac{\omega-35}{\omega-25}\right)^{1/2} = 0.8944$$

$$\omega - 35 = \underbrace{(.8944)^2}_{.8}(\omega - 25)$$

$$(1-.8)\,\omega = 35 - .8(25)$$

$$.2\omega = 15$$

$$\omega = 75$$

Question No. 9:

You are given:

- Mortality follows de Moivre's law. $\Longrightarrow X \sim \text{Uniform on } (0, \omega)$
- $\mathring{e}_{20} = 35$.

Compute q_{30}.

$$T_{20} \sim \text{Uniform on } (0, \omega - 20)$$

$$E(T_{20}) = \frac{1}{2}(\omega - 20) = 35 \Longrightarrow \omega = 90$$

Similarly, $T_{30} \sim \text{Uniform on } (0, 60)$

$$q_{30} = P(T_{30} \leq 1) = \int_0^1 f_{T_{30}}(t)\,dt$$

$$= \int_0^1 \frac{1}{60}\,dt$$

$$q_{30} = \frac{1}{60} \approx \underline{\underline{.0167}}$$

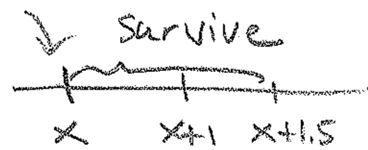
Question No. 10:

You are given:

$$_{k|}q_x = \frac{1}{9}(2k+1), \text{ for } k = 0, 1 \text{ and } 2.$$

Suppose UDD holds between integral ages.

Calculate the probability that a life (x) will survive another 1.5 years.



$$_{0|}q_x = q_x = \frac{1}{9}$$

$$_{1|}q_x = p_x \, q_{x+1} = \frac{8}{9} \, q_{x+1} = \frac{3}{9} \Rightarrow q_{x+1} = \frac{3}{8}$$

Thus,

$$_{1.5}p_x = p_x * {}_{.5}p_{x+1}$$

$$= (1 - q_x) * (1 - .5 \, q_{x+1})$$

$$= \frac{8}{9} * \left(1 - .5 \, \frac{3}{8}\right)$$

$$= \frac{\cancel{8}}{9} * \frac{13}{\cancel{16}\,2} = \frac{13}{18} \approx .7222$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK