MATH 3630
Actuarial Mathematics I
Class Test 1
Wednesday, 30 September 2009
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: ____________________ **Student ID:** ______________

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

Suppose you are given that

$$\ell_x = 10000(100 - x)^2, \text{ for } 0 \leq x \leq 100.$$

Calculate the probability that a person now age 20 will reach retirement age 65.

Question No. 2:

Two friends, Daniel and Brian, are of the same age x. Denote their respective future lifetimes by T_x^{D} and T_x^{B}.

You are given:

- Daniel has a constant force of mortality $\mu_x^{\mathrm{D}} = 0.010$ for all $x > 0$.
- Brian has a constant force of mortality $\mu_x^{\mathrm{B}} = 0.015$ for all $x > 0$.

Calculate the probability that Daniel's future lifetime will exceed Brian's expected future lifetime.

Question No. 3:

You are given the following force of mortality:

$$\mu_x = \begin{cases} 0.02, & \text{for } 0 < x \leq 25 \\ 0.03, & \text{for } x > 25 \end{cases}$$

Calculate the probability that a life (20) will die between the ages of 25 and 35.

Question No. 4:

In a certain population where 3/4 are females and 1/4 are males, the survival function for a female newborn is given by

$$S_X^f(x) = \left(1 - \frac{x}{100}\right)^{1/2}, \text{ for } 0 \le x \le 100,$$

and that for a male newborn is

$$S_X^m(x) = \left(1 - \frac{x}{90}\right)^{1/2}, \text{ for } 0 \le x \le 90.$$

For a randomly selected member of this population, calculate $P(K_{50} = 10)$ where K_{50} is the curtate future lifetime of (50).

Question No. 5:

Suppose you are given the following select-and-ultimate mortality table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	x+3
20	35,600	35,591	35,583	35,572	23
21	35,587	35,576	35,562	35,553	24
22	35,574	35,561	35,547	35,534	25

Assuming UDD between integral ages, calculate ${}_{1|0.4}q_{[20]}$.

Question No. 6:

Assume mortality follows the *Illustrative Life Table*.

Suppose that constant force of mortality assumption holds between integral ages.

Calculate ${}_{1.75|0.75}q_{25}$ and interpret this probability.

Question No. 7:

Suppose you are given the following survival function

$$S_X(x) = e^{-.015x}, \text{ for } x > 0.$$

Calculate e_{30} and interpret this value.

Question No. 8:

Prove that the following holds:

$$\mathring{e}_x = \mathring{e}_{x:\overline{n}|} + {}_np_x \cdot \mathring{e}_{x+n}.$$

Now, suppose you are given:

$$\begin{aligned} \mathring{e}_{20} &= 5.00 \\ \mathring{e}_{20:\overline{5}|} &= 3.75 \\ {}_5p_{20} &= 0.50 \end{aligned}$$

Use the above result to compute $\mathring{e}_{25}$.

Question No. 9:

Justin is currently 40 years old and his mortality follows deMoivre's law with $\omega = 100$.

Justin is contemplating taking up rock climbing as a recreational sport in the coming year. His assumed mortality will be adjusted for the coming year only, so that he will instead have a constant force of mortality of 0.04.

Calculate the percentage decrease in Justin's survival probability for the coming year as a result of taking up rock climbing.

Question No. 10:

You are given the survival function:

$$S_X(x) = \left(1 - \frac{x}{\omega}\right)^r, \text{ for } 0 \le x \le \omega, \text{ and } r > 0.$$

If $\mu_y = 0.10$ and $\mathring{e}_y = 8.75$ for some $0 < y < \omega$, calculate the value of r.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK