MATH 3630
Actuarial Mathematics I
Class Test 1
Wednesday, 24 September 2008
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: ________________________ **Student ID:** _________________

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

You are given that $\mu_x = 0.005$ for all ages $x > 0$.

The probability that (30) will survive another 10 years is p. Given that he survives another 10 years, the probability that he will survive another 10 years is q.

Evaluate p/q and give a brief intuitive explanation of the reasonableness of your answer.

Question No. 2:

You are given:

- $\ell_x = \omega - x$, for $0 \le x \le \omega$; and
- $\mathring{e}_0 = 25$.

Calculate $\mathrm{Var}(T_{20})$ where T_{20} is the future lifetime of (20).

Question No. 3:

Suppose you are given the following force of mortality:

$$\mu_x = \begin{cases} 0.01, & \text{for } 0 < x \leq 30 \\ 0.02, & \text{for } x > 30 \end{cases}$$

Calculate ${}_{20}p_{20}$ and interpret this probability.

Question No. 4:

You are given the following survival function:

$$s_X(x) = \left(1 - \frac{x}{100}\right)^{1/2}, \text{ for } 0 \leq x \leq 100.$$

Calculate $P(K_{20} = 10)$ where K_{20} is the curtate future lifetime of (20). Interpret this probability.

Question No. 5:

You are given the following extracted from a select-and-ultimate mortality table:

x	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}
30	.00439	.00575	.00700
31	.00454	.00598	.00735
32	.00473	.00635	.00790
33	.00511	.00680	.00855
34	.00550	.00738	.00938

The select period is obviously two years.

Calculate ${}_{2|}q_{[30]+1}$.

Question No. 6:

Assume mortality follows the *Illustrative Life Table*.

Suppose that Uniform Distribution of Deaths (UDD) holds between integral ages.

Calculate ${}_{1.5|0.5}q_{65}$.

Question No. 7:

Assume that the force of mortality follows:

$$\mu_x = (1+x)^{-1}, \text{ for } x > 0.$$

Give a simplified expression for ${}_tq_{20}$.

Question No. 8:

Prove that the following holds:

$$\mathring{e}_{x:\overline{m+n}|} = \mathring{e}_{x:\overline{m}|} + {}_m p_x \cdot \mathring{e}_{x+m:\overline{n}|}.$$

Now, suppose you are given:

$$\begin{aligned} \mathring{e}_{20:\overline{4}|} &= 3.7 \\ \mathring{e}_{20:\overline{10}|} &= 8.2 \\ \mathring{e}_{24:\overline{6}|} &= 5.4 \end{aligned}$$

Use the above result to compute the probability that a life (20) will not survive to reach another 4 years.

Question No. 9:

You are given:

$$S_X(x) = \left(1 - \frac{x}{100}\right)^{1/3}, \text{ for } 0 \leq x \leq 100.$$

Calculate $P(T_{30} > 50)$ and interpret this probability.

Question No. 10:

Tony is currently 25 years old and his mortality follows deMoivre's law with $\omega = 100$.

Tony is contemplating taking up paragliding as a recreational sport in the coming year. His assumed mortality will be adjusted for the coming year only, so that he will instead have a constant force of mortality of 0.05.

Calculate the percentage increase in Tony's mortality rate for the coming year as a result of taking up paragliding.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK