MATH 3630

Actuarial Mathematics I

Class Test 1

Wednesday, 24 September 2008

Time Allowed: 1 hour

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: SOLUTIONS Student ID: EMIL

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

You are given that $\mu_x = 0.005$ for all ages $x > 0$.

The probability that (30) will survive another 10 years is p. Given that he survives another 10 years, the probability that he will survive another 10 years is q.

Evaluate p/q and give a brief intuitive explanation of the reasonableness of your answer.

Since μ_x is constant, we have ${}_np_x = e^{-\mu n}$.

$$p = {}_{10}p_{30} = e^{-10\mu} = e^{-.05}$$

$$q = {}_{10}p_{40} = e^{-10\mu} = e^{-.05}$$

Thus, $p = q$ or $\frac{p}{q} = 1$.

Because force of mortality is constant, probabilities of surviving equal number of years will be the same regardless of age. This also comes from the 'memoryless' property of the Exponential.

Question No. 2:

You are given:

- $\ell_x = \omega - x$, for $0 \leq x \leq \omega$; and
- $\mathring{e}_0 = 25$.

Calculate $\mathrm{Var}(T_{20})$ where T_{20} is the future lifetime of (20).

l_x clearly has the property of De Moivre's/Uniform so that

$\mathring{e}_0 = \omega/2 = 25 \Rightarrow \omega = 50$

Thus, $T_{20} \sim$ Uniform or De Moivre's on $(0, 30)$ so that

$$\mathrm{Var}(T_{20}) = \frac{30^2}{12} = 75.$$

Question No. 3:

Suppose you are given the following force of mortality:

$$\mu_x = \begin{cases} 0.01, & \text{for } 0 < x \leq 30 \\ 0.02, & \text{for } x > 30 \end{cases}$$

Calculate ${}_{20}p_{20}$ and interpret this probability.

Consider ${}_t p_{20} = e^{-\int_0^t \mu_{20+s}\, ds}$ where

$$\mu_{20+s} = \begin{cases} .01 & \text{for } 0 < s \leq 10 \\ .02 & \text{for } s > 10 \end{cases}$$

Since we want $t = 20$, we have

$${}_{20}p_{20} = e^{-\int_0^{10} .01\, ds}\, e^{-\int_{10}^{20} .02\, ds}$$

$$= e^{-.1}\, e^{-.2} = e^{-.3} = \underline{\underline{0.7408}}$$

probability a life (20) survives another 20 years.

or the next 20 years.

Question No. 4:

You are given the following survival function:

$$s_X(x) = \left(1 - \frac{x}{100}\right)^{1/2}, \text{ for } 0 \le x \le 100.$$

Calculate $P(K_{20} = 10)$ where K_{20} is the curtate future lifetime of (20). Interpret this probability.

We know distribution of K_x is $P(K_x = k) = {}_{k|}q_x$
$= {}_k p_x \, q_{x+k}$.

Thus, $P(K_{20} = 10) = {}_{10}p_{20} \, q_{30}$

$$= \frac{S_X(30)}{S_X(20)} \left(1 - \frac{S_X(31)}{S_X(30)}\right)$$

$$= \frac{\sqrt{.7}}{\sqrt{.8}} \left(1 - \frac{\sqrt{.69}}{\sqrt{.70}}\right)$$

$$= \underline{.00671}$$

Probability that a life (20) will survive the next 10 completed years.

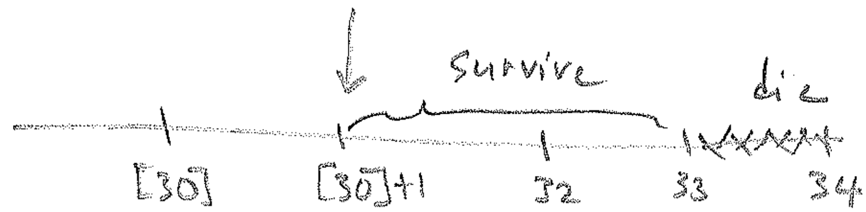
Question No. 5:

You are given the following extracted from a select-and-ultimate mortality table:

x	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}
30	.00439	.00575	.00700
31	.00454	.00598	.00735
32	.00473	.00635	.00790
33	.00511	.00680	.00855
34	.00550	.00738	.00938

The select period is obviously two years.

Calculate ${}_{2|}q_{[30]+1}$.



$$ {}_{2|}q_{[30]+1} = \underbrace{{}_{2}p_{[30]+1}} \cdot q_{\underbrace{[30]+3}_{33}} \quad \text{past select period} $$

$$ p_{[30]+1} * p_{\underbrace{[30]+2}_{32}} $$

$$ = p_{[30]+1} * p_{32} * q_{33} $$

$$ = (1-.00575)(1-.00700)(.00735) $$

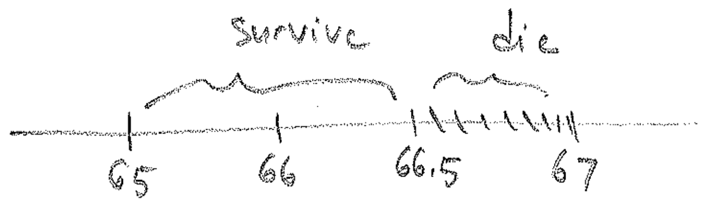
$$ = .00725658 \approx \underline{\underline{.00726}} $$

Question No. 6:

Assume mortality follows the *Illustrative Life Table*.

Suppose that Uniform Distribution of Deaths (UDD) holds between integral ages.

Calculate $_{1.5|0.5}q_{65}$.



First note that

$$_2p_{65} = p_{65} * {}_{.5}p_{66} * \underbrace{{}_{.5}p_{66.5}}_{(1 - {}_{.5}q_{66.5})}$$

$$p_{65}\, p_{66} = p_{65} * {}_{.5}p_{66} - \underbrace{\underbrace{p_{65} * {}_{.5}p_{66} * {}_{.5}q_{66.5}}_{{}_{1.5}p_{65} * {}_{.5}q_{66.5}}}_{{}_{1.5|.5}q_{65}}$$

Solving for $_{1.5|.5}q_{65}$, we get

$$_{1.5|.5}q_{65} = p_{65}\left({}_{.5}p_{66} - p_{66}\right)$$

$$= \left(1 - \frac{21.32}{1000}\right)\left(\left(1 - \frac{1}{2}\left(\frac{23.29}{1000}\right)\right) - \left(1 - \frac{23.29}{1000}\right)\right)$$

$$= \left(1 - \frac{21.32}{1000}\right)\left(\frac{1}{2}\left(\frac{23.29}{1000}\right)\right)$$

$$= .01139672 8$$

Question No. 7:

Assume that the force of mortality follows:

$$\mu_x = (1+x)^{-1}, \text{ for } x > 0.$$

Give a simplified expression for ${}_tq_{20}$.

$${}_tq_{20} = 1 - {}_tp_{20}$$

$$= 1 - e^{-\int_0^t \mu_{20+s}\, ds}$$

$$= 1 - e^{-\int_0^t \frac{1}{21+s}\, ds}$$

$$= 1 - e^{-\left(\log \frac{21+t}{21}\right)} = 1 - \frac{21}{21+t} = \frac{t}{21+t}$$

Question No. 8:

Prove that the following holds:

$$\mathring{e}_{x:\overline{m+n}|} = \mathring{e}_{x:\overline{m}|} + {}_mp_x \cdot \mathring{e}_{x+m:\overline{n}|}.$$

Now, suppose you are given:

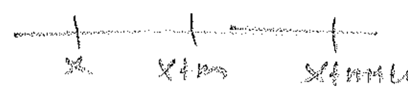
$$\begin{aligned} \mathring{e}_{20:\overline{4}|} &= 3.7 \\ \mathring{e}_{20:\overline{10}|} &= 8.2 \\ \mathring{e}_{24:\overline{6}|} &= 5.4 \end{aligned}$$

Use the above result to compute the probability that a life (20) will not survive to reach another 4 years.

$$\mathring{e}_{x:\overline{m+n}|} = \int_0^{m+n} {}_tp_x\,dt = \underbrace{\int_0^m {}_tp_x\,dt}_{\mathring{e}_{x:\overline{m}|}} + \underbrace{\int_m^{m+n} {}_tp_x\,dt}$$

Change variable
$u = t - m$
$du = dt$

$$= \mathring{e}_{x:\overline{m}|} + \int_0^n \underbrace{{}_{m+u}p_x}_{{}_mp_x \,*\, {}_up_{x+m}}\,du$$



$$= \mathring{e}_{x:\overline{m}|} + {}_mp_x \underbrace{\int_0^n {}_up_{x+m}\,du}_{\mathring{e}_{x+m:\overline{n}|}}$$ which proves the result

applying result for $x=20$, $m=4$, $n=6$, we have

$$\underbrace{\mathring{e}_{20:\overline{10}|}}_{8.2} = \underbrace{\mathring{e}_{20:\overline{4}|}}_{3.7} + {}_4p_{20}\,\underbrace{\mathring{e}_{24:\overline{6}|}}_{5.4}$$

$$\Rightarrow {}_4p_{20} = \frac{5}{6} \Rightarrow \underline{\underline{{}_4q_{20} = \frac{1}{6}}}$$

Question No. 9:

You are given:

$$S_X(x) = \left(1 - \frac{x}{100}\right)^{1/3}, \text{ for } 0 \le x \le 100.$$

Calculate $P(T_{30} > 50)$ and interpret this probability.

$$P(T_{30} > 50) = {}_{50}p_{30} = \frac{S_X(80)}{S_X(30)} = \frac{(1-.8)^{1/3}}{(1-.3)^{1/3}}$$

$$= \left(\frac{2}{7}\right)^{1/3}$$

$$= \underline{\underline{.658633756}}$$

this is the probability that a life (30) will survive the next 50 years.

Question No. 10:

Tony is currently 25 years old and his mortality follows deMoivre's law with $\omega = 100$.

Tony is contemplating taking up paragliding as a recreational sport in the coming year. His assumed mortality will be adjusted for the coming year only, so that he will instead have a constant force of mortality of 0.05.

Calculate the percentage increase in Tony's mortality rate for the coming year as a result of taking up paragliding.

Without paragliding, $T_{25} \sim$ Uniform on $(0, 75)$

$$\therefore q_{25} = \int_0^1 \frac{1}{75}\,dt = \frac{1}{75}$$

With paragliding, constant force of .05

$$\therefore q_{25}^* = \int_0^1 {}_tp_{25}\,\mu_{25+t}\,dt = \int_0^1 .05e^{-.05t}\,dt$$

$$= 1 - e^{-.05}$$

Thus,

percent increase is $\dfrac{q_{25}^*}{q_{25}} - 1 = \dfrac{1 - e^{-.05}}{1/75} - 1$

$$= \underline{\underline{265.78\%}}$$

In effect, approximately Tony's mortality will increase 2.7 times more.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK