Michigan State University

STT 455 - Actuarial Models I

Fall 2014 semester

Homework No. 2

due Friday, 5:00 pm, December 5, 2014

Please follow the instructions below:

Return this page with your signature.

Submit your work to our graduate assistant, Ed Cruz, at C505 Wells.

Write your name and section number at the spaces provided below.

Good luck on your finals. Enjoy the holidays.

Name: SUGGESTED SOLUTIONS **Section:** EMIL

I certify that this is my own work, and that I have not copied the work of another student.

Signature: ______________________ **Date:** ______________________

1. (30 points) For a special life insurance policy issued to (45), you are given:

   - A death benefit of 100,000 is paid at the end of the year of death, if death occurs after age 60.
   - There are no benefits paid, if death occurs before age 60.
   - Premiums are payable monthly, at the beginning of each month, until reaching age 60.

   Premiums are calculated based on the following assumptions:

   - Mortality follows the `Illustrative Life Table`.
   - $i = 0.06$
   - The Woolhouse approximate formula with two terms is used to calculate annuity values.

   (a) [5 points] Calculate the actuarial present value of the death benefit.

   (b) [10 points] Use the equivalence principle to calculate the monthly benefit premium for this policy. Give the annualized benefit premium as well.

   (c) [10 points] Suppose premiums are paid annually, at the beginning of each year, until reaching age 60. Calculate the annual benefit premium.

   (d) [5 points] Explain why there is a difference between the annualized premium calculated in (b) and the annual premium calculated in (c).

(a) $APV(\text{benefit}) = 100{,}000 * {}_{15}E_{45} * A_{60} = 100{,}000\,(.72988)(.51081)$ $(.36913)$

$${}_{15}E_{45} = \underbrace{{}_{5}E_{45} * {}_{10}E_{50}}_{=\,.37283} \qquad = \underline{13{,}762.27}$$

(b) Let P = monthly premium

$$APV(\text{premiums}) = 12P\,\ddot{a}^{(12)}_{45:\overline{15|}}$$

Woolhouse with 2 terms

$$= 12P\left(\ddot{a}_{45:\overline{15|}} - \frac{12-1}{2(12)}\left(1 - {}_{15}E_{45}\right)\right)$$

${}_{15}E_{45} = .37283$

$\ddot{a}_{45:\overline{15|}} = \ddot{a}_{45} - {}_{15}E_{45}\,\ddot{a}_{60}$, with $\ddot{a}_{45} = 14.1121$, $\ddot{a}_{60} = 11.1454$

$$= 12P(9.669308)$$

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Equating the two APV's, we get $P = \dfrac{13762.27}{12(9.669308)} = \underline{118.6079}$

annualized : $12P = \underline{1423.295}$

(c) If paid annually, $APV(\text{premiums}) = P^{*}\ddot{a}_{45:\overline{15|}}$

$$= P^{*}(\ddot{a}_{45} - {}_{15}E_{45}\,\ddot{a}_{60})$$

$$= P^{*}(9.95676)$$

Solving P^{*}, we get

$$P^{*} = \frac{13762.27}{9.95676} = \underline{1382.204}$$

(d) There are mainly two reasons why the annualized premium for premiums paid more frequently than once a year is larger, as it shows in this case:

- Policyholder may die during the year, in which case, the full annualized premium would not have been collected.
- The insurer can reinvest or accrue interest on premiums collected upfront, as in the case when premiums are paid annually in advance.

2. (35 points) For a group of 200 lives with select age $[x]$ and with independent future lifetimes, you are given:

- Each life is to be paid \$1 at the beginning of each year, if alive.
- $A_{[x]} = 0.20$
- ${}^2A_{[x]} = 0.06$
- $i = 0.04$

Y_i is the present value random variable of the payments for life i, $i = 1, 2, \ldots, 200$, so that the present value of the aggregate payments is the sum of these:

$$Y = \sum_{i=1}^{200} Y_i$$

(a) [5 points] Calculate E[Y].

(b) [10 points] Calculate Var[Y].

(c) [15 points] Using the Normal approximation to Y, calculate the initial size of the fund needed in order to be 95% certain of being able to make the payments for these life annuities.

(d) [5 points] If you need an initial fund that ensures you 90% certain of being able to cover the payments, would you expect this to be higher or lower than that in (c)? Explain why. Do not do any calculations.

(a) $E[Y] = \sum_{i=1}^{200} E[Y_i] = \sum_{i=1}^{200} \ddot{a}_{[x]}$ $\qquad \frac{1 - A_{[x]}}{d} = \frac{1 - .20}{.04/1.04} = 20.8$

$= 200(20.8) = \underline{4160}$

(b) $Var[Y] = \sum_{i=1}^{200} Var[Y_i] = \sum_{i=1}^{200} \frac{1}{d^2}\left({}^2A_{[x]} - A_{[x]}^2\right)$

$\underbrace{\frac{1}{(.04/1.04)^2}\left(0.06 - 0.20^2\right)}_{= 13.52}$

$= 200(13.52) = \underline{2704}$

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(c) Let F = required fund

$$\Pr[Y \le F] = 0.95 \Rightarrow \Pr\left[\underbrace{\frac{Y - E[Y]}{\sqrt{Var[Y]}}}_{Z \sim N(0,1)} \le \underbrace{\frac{F - 4160}{\sqrt{2704}}}_{= 1.645}\right] = 0.95$$

$$\frac{F - 4160}{\sqrt{2704}} = 1.645 \Rightarrow F = 4160 + 1.645\sqrt{2704}$$
$$= \underline{4245.54}$$

(d) If you hold a smaller amount than 4245.54, you will have a lower confidence (or probability) of being able to cover your required payments.

3. (35 points) You are given the following survival function for a newborn:

$$S_0(x) = \frac{1}{c}(110 - x)^{2/3},$$

for some constant c.

(a) [5 points] Calculate c so that this survival function is legitimate, and give the limiting age for this survival model.

(b) [5 points] Calculate the expected future lifetime of a newborn.

(c) [5 points] Calculate the probability that a newborn will survive to reach his expected future lifetime.

(d) [5 points] Calculate the probability that a newborn will reach to age 65 but die within 20 years following that.

A special 3-year endowment insurance policy is issued to (50) where:

- The death benefit of 200 is payable at the end of the year of death.
- The endowment is 500 payable at maturity.

(e) [15 points] Assuming this survival model and $i = 0.05$, calculate the actuarial present value of this insurance policy.

(a) $S_0(0) = 1 \Rightarrow \frac{1}{c}(110-0)^{2/3} = 1 \Rightarrow c = 110^{2/3} = 21.54435$

$S_0(\omega) = 0 \Rightarrow \frac{1}{c}(110-\omega)^{2/3} = 0 \Rightarrow \omega = 110$, limiting age

(b) $\mathring{e}_0 = E[T_0] = \int_0^{110} \left(\frac{110-x}{110}\right)^{2/3} dx = \frac{1}{110^{2/3}} \left. \frac{-(110-x)^{5/3}}{5/3} \right|_0^{110}$

$= \frac{1}{\cancel{110^{2/3}}} \cdot \frac{3}{5} \cdot 110^{\cancel{5/3}\,3/3}$

$= \frac{3}{5} \cdot 110 = 66$ years old

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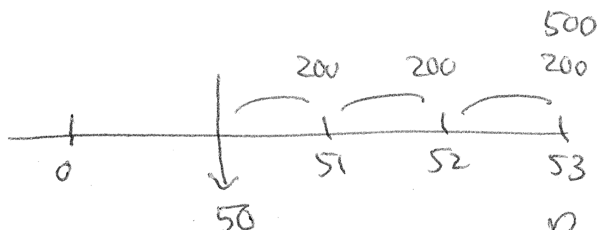
(c) $Pr[T_0 > 66] = S_0(66) = \left(\frac{110-66}{110}\right)^{2/3} = \underline{\underline{0,5428835}}$

(d) $Pr[65 < T_0 \leq 85] = S_0(65) - S_0(85)$

$$= \left(\frac{45}{110}\right)^{2/3} - \left(\frac{25}{110}\right)^{2/3} = \underline{\underline{0.1786594}}$$

(e)



$$\text{APV(insurance)} = 200\left(v q_{50} + v^2 p_{50} q_{51} + v^3 {}_2p_{50} q_{52}\right) + 500 v^3 {}_3p_{50}$$

$p_x = \frac{S_0(x+1)}{S_0(x)}$

${}_tp_x = \frac{S_0(x+t)}{S_0(x)}$

${}_tq_x = 1 - {}_tp_x$

$$= 200\left[\frac{1}{1.05}\left(1-\left(\frac{59}{60}\right)^{2/3}\right) + \frac{1}{1.05^2}\left(\frac{59}{60}\right)^{2/3}\left(1-\left(\frac{58}{59}\right)^{2/3}\right) + \frac{1}{1.05^3}\left(\frac{58}{60}\right)^{2/3}\left(1-\left(\frac{57}{58}\right)^{2/3}\right)\right] + 500\frac{1}{1.05^3}\left(\frac{57}{60}\right)^{2/3}$$

$$= \underline{\underline{419.4355}}$$