

Michigan State University
STT 455 - Actuarial Models I
Fall 2013 semester
Homework No. 2
due Wednesday, 5:00 pm, December 4, 2013

Please follow the instructions below:

Return this page with your signature.

Submit your work to our graduate assistant, Ed Cruz, at C505 Wells.

Write your name and section number at the spaces provided:

Name: _____ **Section:** _____

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____ **Date:** _____

1. (35 points) For a fully discrete 10-year deferred whole life insurance of \$100,000 on (35), you are given:
- Premiums are payable annually, at the beginning of each year, only during the deferral period.
 - There are no death benefits during the deferral period.
 - Mortality follows the Illustrative Life Table.
 - $i = 0.06$
 - $(IA)_{35:\overline{10}|}^1 = 0.10797$
- (a) [5 points] Explain, in words, why insurance policies must have the first premium payable at policy issue.
- (b) [5 points] Write an expression for the loss-at-issue random variable for this policy.
- (c) [10 points] Use the equivalence principle to compute the annual premium.
- (d) [15 points] If an additional death benefit equal to the return of all premiums paid, without interest, is to be paid during the 10-year deferral period, calculate the revised annual premium.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

2. (40 points) On your exact 25th birthday, you have just been hired as an actuarial assistant at *Super Life Insurance Company*. You are entitled to join the company's pension plan which promises to pay:

- a benefit of \$2,000, for life, at the beginning of each month starting at retirement age 65.

Assume that you will work for *Super Life* until retirement. You are given:

- To fund this benefit, *Super Life* will contribute \$500 each year on your birthday, beginning at hire, during your working years.
 - The rest will be paid by you in a fixed amount of C each year on your birthday, beginning at hire, during your working years.
 - Mortality follows the **Illustrative Life Table**
 - $i = 0.06$
 - Mortality assumes that deaths are uniformly distributed over each year of age.
 - For $i = 0.06$, $\alpha(12) = 1.00028$, $\beta(12) = 0.46812$, and $d^{(12)} = 0.05813$.
- (a) [20 points] Calculate C based on the equivalence principle.
- (b) [5 points] Without any calculations, if interest rate is higher than 6%, would you expect your contributions to be higher or lower? Explain.
- (c) [15 points] Suppose you are now age 65 and have reached retirement. You have the option of receiving a lower amount of benefit but with a 10-year guarantee. Calculate your revised monthly benefit.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

3. (25 points) For a fully discrete whole life insurance of \$1,000 issued to (40), you are given:
- There is a first year expense of \$10 and a renewal expense of \$1 each year after the first year.
 - These expenses are paid at the beginning of each year.
 - Mortality follows the Illustrative Life Table.
 - $i = 0.06$
- (a) [20 points] Calculate the expense-loaded annual premium.
- (b) [5 points] Suppose there is an additional expense, called the claim settlement expense, of \$40 payable when the death claim is made. Calculate the new expense-loaded annual premium.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

| x | l_x | $1000q_x$ | \ddot{a}_x | $1000A_x$ | $1000({}^2A_x)$ | $1000{}_5E_x$ | $1000{}_{10}E_x$ | $1000{}_{20}E_x$ | x |
|-----|------------|-----------|--------------|-----------|-----------------|---------------|------------------|------------------|-----|
| 0 | 10,000,000 | 20.42 | 16.8010 | 49.00 | 25.92 | 728.54 | 541.95 | 299.89 | 0 |
| 5 | 9,749,503 | 0.98 | 17.0379 | 35.59 | 8.45 | 743.89 | 553.48 | 305.90 | 5 |
| 10 | 9,705,588 | 0.85 | 16.9119 | 42.72 | 9.37 | 744.04 | 553.34 | 305.24 | 10 |
| 15 | 9,663,731 | 0.91 | 16.7384 | 52.55 | 11.33 | 743.71 | 552.69 | 303.96 | 15 |
| 20 | 9,617,802 | 1.03 | 16.5133 | 65.28 | 14.30 | 743.16 | 551.64 | 301.93 | 20 |
| 21 | 9,607,896 | 1.06 | 16.4611 | 68.24 | 15.06 | 743.01 | 551.36 | 301.40 | 21 |
| 22 | 9,597,695 | 1.10 | 16.4061 | 71.35 | 15.87 | 742.86 | 551.06 | 300.82 | 22 |
| 23 | 9,587,169 | 1.13 | 16.3484 | 74.62 | 16.76 | 742.68 | 550.73 | 300.19 | 23 |
| 24 | 9,576,288 | 1.18 | 16.2878 | 78.05 | 17.71 | 742.49 | 550.36 | 299.49 | 24 |
| 25 | 9,565,017 | 1.22 | 16.2242 | 81.65 | 18.75 | 742.29 | 549.97 | 298.73 | 25 |
| 26 | 9,553,319 | 1.27 | 16.1574 | 85.43 | 19.87 | 742.06 | 549.53 | 297.90 | 26 |
| 27 | 9,541,153 | 1.33 | 16.0873 | 89.40 | 21.07 | 741.81 | 549.05 | 297.00 | 27 |
| 28 | 9,528,475 | 1.39 | 16.0139 | 93.56 | 22.38 | 741.54 | 548.53 | 296.01 | 28 |
| 29 | 9,515,235 | 1.46 | 15.9368 | 97.92 | 23.79 | 741.24 | 547.96 | 294.92 | 29 |
| 30 | 9,501,381 | 1.53 | 15.8561 | 102.48 | 25.31 | 740.91 | 547.33 | 293.74 | 30 |
| 31 | 9,486,854 | 1.61 | 15.7716 | 107.27 | 26.95 | 740.55 | 546.65 | 292.45 | 31 |
| 32 | 9,471,591 | 1.70 | 15.6831 | 112.28 | 28.72 | 740.16 | 545.90 | 291.04 | 32 |
| 33 | 9,455,522 | 1.79 | 15.5906 | 117.51 | 30.63 | 739.72 | 545.07 | 289.50 | 33 |
| 34 | 9,438,571 | 1.90 | 15.4938 | 122.99 | 32.68 | 739.25 | 544.17 | 287.82 | 34 |
| 35 | 9,420,657 | 2.01 | 15.3926 | 128.72 | 34.88 | 738.73 | 543.18 | 286.00 | 35 |
| 36 | 9,401,688 | 2.14 | 15.2870 | 134.70 | 37.26 | 738.16 | 542.11 | 284.00 | 36 |
| 37 | 9,381,566 | 2.28 | 15.1767 | 140.94 | 39.81 | 737.54 | 540.92 | 281.84 | 37 |
| 38 | 9,360,184 | 2.43 | 15.0616 | 147.46 | 42.55 | 736.86 | 539.63 | 279.48 | 38 |
| 39 | 9,337,427 | 2.60 | 14.9416 | 154.25 | 45.48 | 736.11 | 538.22 | 276.92 | 39 |
| 40 | 9,313,166 | 2.78 | 14.8166 | 161.32 | 48.63 | 735.29 | 536.67 | 274.14 | 40 |
| 41 | 9,287,264 | 2.98 | 14.6864 | 168.69 | 52.01 | 734.40 | 534.99 | 271.12 | 41 |
| 42 | 9,259,571 | 3.20 | 14.5510 | 176.36 | 55.62 | 733.42 | 533.14 | 267.85 | 42 |
| 43 | 9,229,925 | 3.44 | 14.4102 | 184.33 | 59.48 | 732.34 | 531.12 | 264.31 | 43 |
| 44 | 9,198,149 | 3.71 | 14.2639 | 192.61 | 63.61 | 731.17 | 528.92 | 260.48 | 44 |
| 45 | 9,164,051 | 4.00 | 14.1121 | 201.20 | 68.02 | 729.88 | 526.52 | 256.34 | 45 |
| 46 | 9,127,426 | 4.31 | 13.9546 | 210.12 | 72.72 | 728.47 | 523.89 | 251.88 | 46 |
| 47 | 9,088,049 | 4.66 | 13.7914 | 219.36 | 77.73 | 726.93 | 521.03 | 247.08 | 47 |
| 48 | 9,045,679 | 5.04 | 13.6224 | 228.92 | 83.06 | 725.24 | 517.91 | 241.93 | 48 |
| 49 | 9,000,057 | 5.46 | 13.4475 | 238.82 | 88.73 | 723.39 | 514.51 | 236.39 | 49 |
| 50 | 8,950,901 | 5.92 | 13.2668 | 249.05 | 94.76 | 721.37 | 510.81 | 230.47 | 50 |
| 51 | 8,897,913 | 6.42 | 13.0803 | 259.61 | 101.15 | 719.17 | 506.78 | 224.15 | 51 |
| 52 | 8,840,770 | 6.97 | 12.8879 | 270.50 | 107.92 | 716.76 | 502.40 | 217.42 | 52 |
| 53 | 8,779,128 | 7.58 | 12.6896 | 281.72 | 115.09 | 714.12 | 497.64 | 210.27 | 53 |
| 54 | 8,712,621 | 8.24 | 12.4856 | 293.27 | 122.67 | 711.24 | 492.47 | 202.70 | 54 |
| 55 | 8,640,861 | 8.96 | 12.2758 | 305.14 | 130.67 | 708.10 | 486.86 | 194.72 | 55 |
| 56 | 8,563,435 | 9.75 | 12.0604 | 317.33 | 139.11 | 704.67 | 480.79 | 186.32 | 56 |
| 57 | 8,479,908 | 10.62 | 11.8395 | 329.84 | 147.99 | 700.93 | 474.22 | 177.53 | 57 |
| 58 | 8,389,826 | 11.58 | 11.6133 | 342.65 | 157.33 | 696.85 | 467.12 | 168.37 | 58 |
| 59 | 8,292,713 | 12.62 | 11.3818 | 355.75 | 167.13 | 692.41 | 459.46 | 158.87 | 59 |
| 60 | 8,188,074 | 13.76 | 11.1454 | 369.13 | 177.41 | 687.56 | 451.20 | 149.06 | 60 |
| 61 | 8,075,403 | 15.01 | 10.9041 | 382.79 | 188.17 | 682.29 | 442.31 | 139.00 | 61 |
| 62 | 7,954,179 | 16.38 | 10.6584 | 396.70 | 199.41 | 676.56 | 432.77 | 128.75 | 62 |
| 63 | 7,823,879 | 17.88 | 10.4084 | 410.85 | 211.13 | 670.33 | 422.54 | 118.38 | 63 |
| 64 | 7,683,979 | 19.52 | 10.1544 | 425.22 | 223.34 | 663.56 | 411.61 | 107.97 | 64 |
| 65 | 7,533,964 | 21.32 | 9.8969 | 439.80 | 236.03 | 656.23 | 399.94 | 97.60 | 65 |

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

| x | l_x | $1000q_x$ | \ddot{a}_x | $1000A_x$ | $1000({}^2A_x)$ | $1000{}_5E_x$ | $1000{}_{10}E_x$ | $1000{}_{20}E_x$ | x |
|-----|-----------|-----------|--------------|-----------|-----------------|---------------|------------------|------------------|-----|
| 66 | 7,373,338 | 23.29 | 9.6362 | 454.56 | 249.20 | 648.27 | 387.53 | 87.37 | 66 |
| 67 | 7,201,635 | 25.44 | 9.3726 | 469.47 | 262.83 | 639.66 | 374.36 | 77.38 | 67 |
| 68 | 7,018,432 | 27.79 | 9.1066 | 484.53 | 276.92 | 630.35 | 360.44 | 67.74 | 68 |
| 69 | 6,823,367 | 30.37 | 8.8387 | 499.70 | 291.46 | 620.30 | 345.77 | 58.54 | 69 |
| 70 | 6,616,155 | 33.18 | 8.5693 | 514.95 | 306.42 | 609.46 | 330.37 | 49.88 | 70 |
| 71 | 6,396,609 | 36.26 | 8.2988 | 530.26 | 321.78 | 597.79 | 314.27 | 41.86 | 71 |
| 72 | 6,164,663 | 39.62 | 8.0278 | 545.60 | 337.54 | 585.25 | 297.51 | 34.53 | 72 |
| 73 | 5,920,394 | 43.30 | 7.7568 | 560.93 | 353.64 | 571.81 | 280.17 | 27.96 | 73 |
| 74 | 5,664,051 | 47.31 | 7.4864 | 576.24 | 370.08 | 557.43 | 262.31 | 22.19 | 74 |
| 75 | 5,396,081 | 51.69 | 7.2170 | 591.49 | 386.81 | 542.07 | 244.03 | 17.22 | 75 |
| 76 | 5,117,152 | 56.47 | 6.9493 | 606.65 | 403.80 | 525.71 | 225.46 | 13.04 | 76 |
| 77 | 4,828,182 | 61.68 | 6.6836 | 621.68 | 421.02 | 508.35 | 206.71 | 9.61 | 77 |
| 78 | 4,530,360 | 67.37 | 6.4207 | 636.56 | 438.42 | 489.97 | 187.94 | 6.88 | 78 |
| 79 | 4,225,163 | 73.56 | 6.1610 | 651.26 | 455.95 | 470.57 | 169.31 | 4.77 | 79 |
| 80 | 3,914,365 | 80.30 | 5.9050 | 665.75 | 473.59 | 450.19 | 151.00 | 3.19 | 80 |
| 81 | 3,600,038 | 87.64 | 5.6533 | 680.00 | 491.27 | 428.86 | 133.19 | 2.05 | 81 |
| 82 | 3,284,542 | 95.61 | 5.4063 | 693.98 | 508.96 | 406.62 | 116.06 | 1.27 | 82 |
| 83 | 2,970,496 | 104.28 | 5.1645 | 707.67 | 526.60 | 383.57 | 99.81 | 0.75 | 83 |
| 84 | 2,660,734 | 113.69 | 4.9282 | 721.04 | 544.15 | 359.79 | 84.59 | 0.42 | 84 |
| 85 | 2,358,246 | 123.89 | 4.6980 | 734.07 | 561.57 | 335.40 | 70.56 | 0.22 | 85 |
| 86 | 2,066,090 | 134.94 | 4.4742 | 746.74 | 578.80 | 310.56 | 57.83 | 0.11 | 86 |
| 87 | 1,787,299 | 146.89 | 4.2571 | 759.03 | 595.79 | 285.44 | 46.50 | 0.05 | 87 |
| 88 | 1,524,758 | 159.81 | 4.0470 | 770.92 | 612.51 | 260.21 | 36.61 | 0.02 | 88 |
| 89 | 1,281,083 | 173.75 | 3.8442 | 782.41 | 628.92 | 235.11 | 28.17 | 0.01 | 89 |
| 90 | 1,058,491 | 188.77 | 3.6488 | 793.46 | 644.96 | 210.36 | 21.13 | 0.00 | 90 |
| 91 | 858,676 | 204.93 | 3.4611 | 804.09 | 660.61 | 186.21 | 15.41 | 0.00 | 91 |
| 92 | 682,707 | 222.27 | 3.2812 | 814.27 | 675.83 | 162.90 | 10.91 | 0.00 | 92 |
| 93 | 530,959 | 240.86 | 3.1091 | 824.01 | 690.59 | 140.69 | 7.47 | 0.00 | 93 |
| 94 | 403,072 | 260.73 | 2.9450 | 833.30 | 704.86 | 119.79 | 4.93 | 0.00 | 94 |
| 95 | 297,981 | 281.91 | 2.7888 | 842.14 | 718.61 | 100.43 | 3.13 | 0.00 | 95 |
| 96 | 213,977 | 304.45 | 2.6406 | 850.53 | 731.83 | 82.78 | 1.90 | 0.00 | 96 |
| 97 | 148,832 | 328.34 | 2.5002 | 858.48 | 744.50 | 66.97 | 1.10 | 0.00 | 97 |
| 98 | 99,965 | 353.60 | 2.3676 | 865.99 | 756.60 | 53.09 | 0.60 | 0.00 | 98 |
| 99 | 64,617 | 380.20 | 2.2426 | 873.06 | 768.13 | 41.14 | 0.31 | 0.00 | 99 |
| 100 | 40,049 | 408.12 | 2.1252 | 879.70 | 779.08 | 31.12 | 0.15 | 0.00 | 100 |
| 101 | 23,705 | 437.28 | 2.0152 | 885.93 | 789.44 | 22.91 | 0.07 | 0.00 | 101 |
| 102 | 13,339 | 467.61 | 1.9123 | 891.76 | 799.21 | 16.37 | 0.03 | 0.00 | 102 |
| 103 | 7,101 | 498.99 | 1.8164 | 897.19 | 808.41 | 11.33 | 0.01 | 0.00 | 103 |
| 104 | 3,558 | 531.28 | 1.7273 | 902.23 | 817.02 | 7.56 | 0.00 | 0.00 | 104 |
| 105 | 1,668 | 564.29 | 1.6447 | 906.90 | 825.06 | 4.86 | 0.00 | 0.00 | 105 |
| 106 | 727 | 597.83 | 1.5685 | 911.22 | 832.53 | 2.99 | 0.00 | 0.00 | 106 |
| 107 | 292 | 631.64 | 1.4984 | 915.19 | 839.46 | 1.76 | 0.00 | 0.00 | 107 |
| 108 | 108 | 665.45 | 1.4341 | 918.82 | 845.84 | 0.98 | 0.00 | 0.00 | 108 |
| 109 | 36 | 698.97 | 1.3755 | 922.14 | 851.69 | 0.52 | 0.00 | 0.00 | 109 |
| 110 | 11 | 731.87 | 1.3223 | 925.15 | 857.04 | 0.26 | 0.00 | 0.00 | 110 |