Michigan State University
STT 455 - Actuarial Models I
Fall 2013 semester
Homework No. 2
due Wednesday, 5:00 pm, December 4, 2013

Please follow the instructions below:

Return this page with your signature.

Submit your work to our graduate assistant, Ed Cruz, at C505 Wells.

Write your name and section number at the spaces provided:

Name: Suggested Solutions **Section:** _____

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____ **Date:** _____

1. (35 points) For a fully discrete 10-year deferred whole life insurance of \$100,000 on (35), you are given:

   - Premiums are payable annually, at the beginning of each year, only during the deferral period.
   - There are no death benefits during the deferral period.
   - Mortality follows the `Illustrative Life Table`.
   - $i = 0.06$
   - $(IA)^1_{35:\overline{10}|} = .10797$

   (a) [5 points] Explain, in words, why insurance policies must have the first premium payable at policy issue.

   (b) [5 points] Write an expression for the loss-at-issue random variable for this policy.

   (c) [10 points] Use the equivalence principle to compute the annual premium.

   (d) [15 points] If an additional death benefit equal to the return of all premiums paid, without interest, is to be paid during the 10-year deferral period, calculate the revised annual premium.

(a) If no premium is paid at policy issue, a person could buy the policy and then withdraw just before the first premium is due. This gives a window for the person to have insurance without paying for it!

(b) Let K be the curtate future lifetime of (35),

$$L_0 = PVFB_0 - PVFP_0 = \begin{cases} 0 - P\ddot{a}_{\overline{K+1}|}, & K < 10 \\ v^{K+1} - P\ddot{a}_{\overline{n}|}, & K \geq 10 \end{cases}$$

$100{,}000$ *

(c) $APV(FP_0) = APV(FB_0) \Rightarrow P\ddot{a}_{35:\overline{10}|} = 100{,}000\,{}_{10}E_{35}\,A_{45}$

Solving for P, we get $P = 100{,}000\,\dfrac{{}_{10}E_{35}\,A_{45}}{\ddot{a}_{35:\overline{10}|}}$

$$P = 100{,}000 \frac{{}_{10}E_{35}A_{45}}{\ddot{a}_{35} - {}_{10}E_{35}\ddot{a}_{45}} = \frac{100{,}000\,(.54318)(.20120)}{15.3926 - (.54318)(14.1121)}$$

$$= 100{,}000 * \frac{0.1092878}{7.72719} = 1414.328$$

(d) $APV(FP_0) = P\,\ddot{a}_{35:\overline{10}|}$

$APV(FB_0) = P\underbrace{(IA)^{1}_{35:\overline{10}|}}_{.10797} + 100{,}000\,{}_{10}E_{35}A_{45}$

$$P = \frac{60{,}000\,{}_{10}E_{35}A_{45}}{\ddot{a}_{35:\overline{10}|} - (IA)^{1}_{35:\overline{10}|}}$$

$$= \frac{100{,}000\,(.54318)(.20120)}{7.72719 - .10797}$$

$$= 100{,}000 * \frac{0.1092878}{7.61922} = 1434.37$$

2. (40 points) On your exact 25th birthday, you have just been hired as an actuarial assistant at *Super Life Insurance Company.* You are entitled to join the company's pension plan which promises to pay:

   - a benefit of \$2,000, for life, at the beginning of each month starting at retirement age 65.

   Assume that you will work for *Super Life* until retirement. You are given:

   - To fund this benefit, *Super Life* will contribute \$500 each year on your birthday, beginning at hire, during your working years.
   - The rest will be paid by you in a fixed amount of C each year on your birthday, beginning at hire, during your working years.
   - Mortality follows the `Illustrative Life Table`
   - $i = 0.06$
   - Mortality assumes that deaths are uniformly distributed over each year of age.
   - For $i = 0.06$, $\alpha(12) = 1.00028$ ~~and~~ $\beta(12) = 0.46812$, and $d^{(12)} = .05813$

   (a) [20 points] Calculate C based on the equivalence principle.

   (b) [5 points] Without any calculations, if interest rate is higher than 6%, would you expect your contributions to be higher or lower? Explain.

   (c) [15 points] Suppose you are now age 65 and have reached retirement. You have the option of receiving a lower amount of benefit but with a 10-year guarantee. Calculate your revised monthly benefit.

(a) APV(contributions) = APV(pension benefits)

$20E_{25} * 20E_{45}$

$(.29873)(.25634) = .07657695$

$(C+500)\,\ddot{a}_{25:\overline{40}|} = 12(2000)\,{}_{40}E_{25}\,\ddot{a}^{(12)}_{65}$

where $\ddot{a}_{25:\overline{40}|} = \ddot{a}_{25} - {}_{40}E_{25}\,\ddot{a}_{65}$

$= 16.2242 - (.07657645)(9.8969)$

$= 15.46633$

and $\ddot{a}^{(12)}_{65} = \alpha(12)\,\ddot{a}_{65} - \beta(12)$

$= 1.00028(9.8969) - 0.46812$

$= 9.431551$

Solving for C, we get

$$C = \frac{12(2000)(.07657645)(9.431551)}{15.46633} - 500$$

$$= \text{[redacted]} \quad 620.7334$$

(b) Contributions plus interest earned fund the benefits. If the interest rate is higher, contributions will be lower!

(c)

Let B = revised benefit

$$12(2000)\,\ddot{a}^{(12)}_{65} = 12B\left(\ddot{a}^{(12)}_{\overline{10}|} + {}_{10}E_{65}\,\ddot{a}^{(12)}_{75}\right)$$

.39994

7.2170

$$\ddot{a}^{(12)}_{\overline{10}|} = \frac{1-v^{10}}{d^{(12)}} = 7.596856$$

$$\ddot{a}^{(12)}_{75} = \alpha(12)\,\ddot{a}_{75} - \beta(12) = 6.750901$$

$$B = \frac{2000(9.431551)}{7.596856 + .39994(6.750901)}$$

$$= \frac{18863.1}{10.29681} = 1,831.936$$ ✓ This is correct!

3. (25 points) For a fully discrete whole life insurance of \$1,000 issued to (40), you are given:

   - There is a first year expense of \$10 and a renewal expense of \$1 each year after the first year.
   - These expenses are paid at the beginning of each year.
   - Mortality follows the `Illustrative Life Table`.
   - $i = 0.06$

   (a) [20 points] Calculate the expense-loaded annual premium.

   (b) [5 points] Suppose there is an additional expense, called the claim settlement expense, of \$40 payable when the death claim is made. Calculate the new expense-loaded annual premium.

(a) $APV(FP_0) = P\ddot{a}_{40}$

$APV(FB_0) = 1000\, A_{40}$

$APV(FE_0) = 9 + \ddot{a}_{40}$

Using equivalence principle, the expense-loaded annual premium is

$$P = \frac{1000A_{40} + 9 + \ddot{a}_{40}}{\ddot{a}_{40}} = \frac{1000(.16132) + 9 + 14.8166}{14.8166}$$

$$= 12.49521$$

(b) The claim settlement will simply increase $APV(FE_0)$ by

$40A_{40} = 40(.16132) = 6.4528$, so that

$$P = 12.49521 + \frac{6.4528}{14.8166} = 12.93072$$

ES 12/10/2013

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000{}_5E_x$	$1000{}_{10}E_x$	$1000{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000({}^2A_x)$	$1000{}_5E_x$	$1000{}_{10}E_x$	$1000{}_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110