Michigan State University

STT 455 - Actuarial Models I
Fall 2014 semester
Homework No. 1
due Friday, 5:00 pm, September 19, 2014

Please follow the instructions below:

Return this page with your signature.

Submit your work to our graduate assistant, Ed Cruz, at C505 Wells.

Write your name and section number at the spaces provided:

Name: _____ **Section:** _____

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____ **Date:** _____

1. (35 points) Suppose that the future lifetime of a newborn follows the survival function

$$S_0(t) = \left(\frac{105 - t}{105}\right)^{1/3}, \quad \text{for } 0 < t \leq 105.$$

   (a) [15 points] Explain why this is a valid survival function.

   (b) [10 points] Calculate $\mathrm{E}(T_0) = \mathring{e}_0$

   (c) [5 points] Calculate ${}_{10}q_{30}$ and interpret this value.

   (d) [5 points] Calculate the probability that (40) will die between ages 65 and 75.

2. (25 points) Suppose that T_0 follows a constant force with density

$$f_0(t) = \frac{1}{100} e^{-t/100}, \quad \text{for } t > 0.$$

   (a) [8 points] Explain why T_x, for any age $x > 0$, follows a constant force with similar density as

$$f_x(t) = \frac{1}{100} e^{-t/100}, \quad \text{for } t > 0.$$

   (b) [5 points] Calculate $\mathrm{E}(T_x) = \mathring{e}_x$.

   (c) [7 points] Calculate e_x.

   (d) [5 points] Explain briefly why (b) is different from (c).

3. (40 points) You are given the force of mortality:

$$\mu_x = a + e^{bx}$$

where a and b are positive constants. In addition, you are given the following values:

$$p_0 = 0.30068 \qquad p_1 = 0.26920 \qquad p_2 = 0.23822$$

(a) [15 points] Show that the following expression is true:

$${}_tp_x = e^{-at} \exp\left[-\frac{e^{bx}}{b}(e^{bt} - 1)\right]$$

(b) [20 points] Calculate the constants a and b. HINT: Calculate the expression:

$$\frac{\log(p_2) - \log(p_1)}{\log(p_1) - \log(p_0)}$$

See also DHW, Exercise 2.11.

(c) [5 points] Calculate μ_{45}.