Actuarial Applications of a Hierarchical Insurance Claims Model

joint work with E.W. Frees and Peng Shi, both from the University of Wisconsin - Madison

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The hierarchical insurance claims model

- Traditional to predict/estimate insurance claims distributions:

$$\text{Cost of Claims} = \text{Frequency} \times \text{Severity}$$

- Joint density of the aggregate loss can be decomposed as:

$$\begin{aligned} f(N, \mathbf{M}, \mathbf{y}) &= f(N) \times f(\mathbf{M}|N) \times f(\mathbf{y}|N, \mathbf{M}) \\ \text{joint} &= \text{frequency} \times \text{conditional claim-type} \\ &\quad \times \text{conditional severity}, \end{aligned}$$

  where $f(N, \mathbf{M}, \mathbf{y})$ denotes the joint aggregate loss density and is equal to the product of the frequency, conditional claim-type, and conditional severity components.

- Such natural decomposition allows us to investigate/model each component separately.
- Frees and Valdez (2008), Hierarchical Insurance Claims Modeling, *Journal of the American Statistical Association*, to appear.

Salient features of the hierarchical model

- Allows for risk rating factors to be used as explanatory variables that predict both the frequency and the multivariate severity components.
- Helps capture the long-tail nature of the claims distribution through the GB2 distribution model.
- Provides for a “two-part” distribution of losses - when a claim occurs, not necessary that all possible types of losses are realized.
- Allows to capture possible dependencies of claims among the various types through a t-copula specification.

Risk factor rating system

- Insurers adopt "risk factor rating system" in establishing premiums for motor insurance.
- Some risk factors considered:
  - vehicle characteristics: make/brand/model, engine capacity, year of make (or age of vehicle), price/value
  - driver characteristics: age, sex, occupation, driving experience, claim history
  - other characteristics: what to be used for (private, corporate, commercial, hire), type of coverage
- The "no claims discount" (NCD) system:
  - rewards for safe driving
  - discount upon renewal of policy ranging from 0 to 50%, depending on the number of years of zero claims.
- These risk factors/characteristics help explain the heterogeneity among the individual policyholders.

The purpose of this applications paper

- Analyze the risk profile of either a single individual policy, or a portfolio of these policies.
- Our paper focuses on three different types of actuarial applications:
  - Calculation of the predictive mean of losses for individual risk rating.
    - allows the actuary to differentiate premium rates based on policyholder characteristics.
    - quantifies the non-linear effects of coverage modifications like deductibles, policy limits, and coinsurance.
    - possible “unbundling” of contracts.
  - Evaluating the predictive distribution of portfolio of policies.
    - assists insurers in determining appropriate economic capital.
    - measures used are standard: value-at-risk (VaR) and conditional tail expectation (CTE).
  - Examining effects of several reinsurance treaties:
    - quota share versus excess-of-loss arrangements.
    - analysis of retention limits at both the policy and portfolio level.

The literature on claims frequency/severity

- There is vast literature on modeling claims frequency and severity:
  - Klugman, Panjer and Willmot (2004); Boucher and Denuit (2006) - more recent.
  - Kahane and Levy (1975) - some of the earlier work.
  - Coutts (1984) claims there is more extensive literature on frequency modeling.
- Applications to motor insurance:
  - Brockman and Wright (1992) - good early overview.
  - Renshaw (1994) - uses GLM.
  - Most papers use grouped data, unlike the use of the level of details in our papers.
- More modern statistical approaches:
  - Pinquet (1997, 1998) - cross-sectional data, policyholders over time.
    - considered 2 lines of business: claims at fault and not at fault; allowed correlation using a bivariate Poisson for frequency; severity models used were lognormal and gamma.

The data observed

- Model is calibrated based on detailed, micro-level automobile insurance records over nine years [1993 to 2001] of a randomly selected Singapore insurer.
- Information was extracted from three databases:
  - policy file
  - claims file
  - payment file
- The data observed is a registered vehicle insured i over time t (year). For each observational unit $\{it\}$, the observable data consists of:
  - number of claims within a year: N_{it}
  - type of claim, available for each claim: k for $k = 1, 2, 3$
  - the loss amount, for each claim: y_{itk} for $t = 1, \ldots, T_i$, $i = 1, \ldots, n$ and for type $k = 1, 2, 3$.
  - exposure: e_{it}
  - vehicle characteristics: described by the vector $\mathbf{x}_{it}$
- The data available therefore consist of

$$\{e_{it}, \mathbf{x}_{it}, N_{it}, y_{itk},\ k = 1, 2, 3,\ t = 1, \ldots, T_i,\ i = 1, \ldots, n\}.$$

Types of losses

- When a claim is made, possible to have one or a combination of three (3) types of losses:
  1. losses for injury to a party other than the insured y_{ij1} - “injury”;
  2. losses for damages to the insured, including injury, property damage, fire and theft y_{ij2} - “own damage”; and
  3. losses for property damage to a party other than the insured y_{ij3} - “third party property”.

Table 2. Value of M, by Claim Type

Value of M	1	2	3	4	5	6	7
Claim by Combination Observed	(y_1)	(y_2)	(y_3)	(y_1, y_2)	(y_1, y_3)	(y_2, y_3)	(y_1, y_2, y_3)

Covariates used

- Year: the calendar year - 1993-2001; treated as continuous variable.
- Vehicle Type: automotive (A) or others (O).
- Vehicle Age: in years, grouped into 6 categories -
  - 0, 1-2, 3-5, 6-10, 11-15, <=16.
- Vehicle Capacity: in cubic capacity.
- Gender: male (M) or female (F).
- Age: in years, grouped into 7 categories -
  - ages >=21, 22-25, 26-35, 36-45, 46-55, 56-65, <=66.
- The NCD applicable for the calendar year - 0%, 10%, 20%, 30%, 40%, and 50%.

Random effects Negative Binomial count model

- Let $\lambda_{it} = e_{it} \exp\left(\mathbf{x}'_{\lambda,it}\beta_\lambda\right)$ be the conditional mean parameter for the $\{it\}$ observational unit, where
  - $\mathbf{x}_{\lambda,it}$ is a subset of $\mathbf{x}_{it}$ representing the variables needed for frequency modeling.
- Negative binomial distribution model with parameters p and r:
  - $\Pr(N = k|r, p) = \binom{k+r-1}{r-1} p^r (1-p)^k.$
  - Here, $\sigma = r^{-1}$ is the dispersion parameter and
  - $p = p_{it}$ is related to the mean through

$$(1 - p_{it})/p_{it} = \lambda_{it}\sigma = e_{it}\exp(\mathbf{x}'_{\lambda,it}\beta_\lambda)\sigma.$$

The conditional claim type component

- Certain characteristics help to describe the types of claims that arise.
- To explain this feature, we use the multinomial logit of the form
$$\Pr(M=m)=\frac{\exp(V_m)}{\sum_{s=1}^{7}\exp(V_s)},$$
where $V_m = V_{it,m} = \mathbf{x}'_{M,it}\beta_{M,m}$.
- For our purposes, the covariates in $\mathbf{x}_{M,it}$ do not depend on the accident number j nor on the claim type m, but we do allow the parameters to depend on m.
- Such has been proposed in Terza and Wilson (1990).

The marginals for the severity component

- We are particularly interested in accommodating the long-tail nature of claims.
- We use the generalized beta of the second kind (GB2) for each claim type with density

$$f(y) = \frac{\exp(\alpha_1 z)}{y|\sigma| B(\alpha_1, \alpha_2) \left[1 + \exp(z)\right]^{\alpha_1 + \alpha_2}},$$

  where $z = (\ln y - \mu)/\sigma$.
- μ is a location parameter, σ is a scale parameter and α_1 and α_2 are shape parameters.
- With four parameters, the distribution has great flexibility for fitting heavy tailed data.
- Many distributions useful for fitting long-tailed distributions can be written as special or limiting cases of the GB2 distribution; see, for example, McDonald and Xu (1995).

GB2 regression

- We allowed scale and shape parameters to vary by type and thus consider α_{1k}, α_{2k} and σ_k for $k = 1, 2, 3$.
- Despite its prominence, there are relatively few applications that use the GB2 in a regression context:
  - McDonald and Butler (1990) used the GB2 with regression covariates to examine the duration of welfare spells.
  - Beirlant et al. (1998) demonstrated the usefulness of the Burr XII distribution, a special case of the GB2 with $\alpha_1 = 1$, in regression applications.
  - Sun et al. (2006) used the GB2 in a longitudinal data context to forecast nursing home utilization.
- We parameterize the location parameter as $\mu_{ik} = \mathbf{x}_{ik}' \beta_k$:
  - Interpretability of parameters.
  - Here then $\beta_{k,j} = \partial \ln \mathrm{E}\left(Y \,|\, \mathbf{x}\right) / \partial x_j$, meaning that we may interpret the regression coefficients as proportional changes.

Dependencies among claim types

- We use a parametric copula (in particular, the t copula).
- Suppressing the $\{i\}$ subscript, we can express the joint distribution of claims (y_1, y_2, y_3) as

$$\mathrm{F}(y_1, y_2, y_3) = \mathrm{H}\left(\mathrm{F}_1(y_1), \mathrm{F}_2(y_2), \mathrm{F}_3(y_3)\right).$$

- Here, the marginal distribution of y_k is given by $\mathrm{F}_k(\cdot)$ and $\mathrm{H}(\cdot)$ is the copula.
- Modeling the joint distribution of the simultaneous occurrence of the claim types, when an accident occurs, provides the unique feature of our work.
- Some references are: Frees and Valdez (1998), Nelsen (1999).

Parameter estimates of the hierarchical model

- Appendix A.1 provides the fitted models for each of the components in the hierarchical model:
  - Table A.1 provides the estimates for the frequency component.
  - Table A.2 provides the estimates for the conditional claim type component.
  - Table A.4 provides the estimates for the severity component.

The fitted frequency model

Table A.1. Fitted Negative Binomial Model

Parameter	Estimate	Standard Error
intercept	-2.275	0.730
year	0.043	0.004
automobile	-1.635	0.082
vehicle age 0	0.273	0.739
vehicle age 1-2	0.670	0.732
vehicle age 3-5	0.482	0.732
vehicle age 6-10	0.223	0.732
vehicle age 11-15	0.084	0.772
automobile*vehicle age 0	0.613	0.167
automobile*vehicle age 1-2	0.258	0.139
automobile*vehicle age 3-5	0.386	0.138
automobile*vehicle age 6-10	0.608	0.138
automobile*vehicle age 11-15	0.569	0.265
automobile*vehicle age $\gg$16	0.930	0.677
vehicle capacity	0.116	0.018
automobile*NCD 0	0.748	0.027
automobile*NCD 10	0.640	0.032
automobile*NCD 20	0.585	0.029
automobile*NCD 30	0.563	0.030
automobile*NCD 40	0.482	0.032
automobile*NCD 50	0.347	0.021
automobile*age $\ll$21	0.955	0.431
automobile*age 22-25	0.843	0.105
automobile*age 26-35	0.657	0.070
automobile*age 36-45	0.546	0.070
automobile*age 46-55	0.497	0.071
automobile*age 56-65	0.427	0.073
automobile*age $\gg$66	0.438	0.087
automobile*male	-0.252	0.042
automobile*female	-0.383	0.043
r	2.167	0.195

The fitted conditional claim type model

Table A.2. Fitted Multi Logit Model

	Parameter Estimates				
Category(M)	intercept	year	vehicle age $\gg$6	non-automobile	automobile*age $\gg$46
1	1.194	-0.142	0.084	0.262	0.128
2	4.707	-0.024	-0.024	-0.153	0.082
3	3.281	-0.036	0.252	0.716	-0.201
4	1.052	-0.129	0.037	-0.349	0.338
5	-1.628	0.132	0.132	-0.008	0.330
6	3.551	-0.089	0.032	-0.259	0.203

The fitted conditional severity model

Table A.4. Fitted Severity Model by Copulas						
	Types of Copula					
Parameter	Independence		Normal Copula		t-Copula	
	Estimate	Standard Error	Estimate	Standard Error	Estimate	Standard Error
Third Party Injury						
σ_1	0.225	0.020	0.224	0.044	0.232	0.079
α_{11}	69.958	28.772	69.944	63.267	69.772	105.245
α_{21}	392.362	145.055	392.372	129.664	392.496	204.730
intercept	34.269	8.144	34.094	7.883	31.915	5.606
Own Damage						
σ_2	0.671	0.007	0.670	0.002	0.660	0.004
α_{12}	5.570	0.151	5.541	0.144	5.758	0.103
α_{22}	12.383	0.628	12.555	0.277	13.933	0.750
intercept	1.987	0.115	2.005	0.094	2.183	0.112
year	-0.016	0.006	-0.015	0.006	-0.013	0.006
vehicle capacity	0.116	0.031	0.129	0.022	0.144	0.012
vehicle age $\ll 5$	0.107	0.034	0.106	0.031	0.107	0.003
automobile*NCD 0-10	0.102	0.029	0.099	0.039	0.087	0.031
automobile*age 26-55	-0.047	0.027	-0.042	0.044	-0.037	0.005
automobile*age $\gg 56$	0.101	0.050	0.080	0.018	0.084	0.050
Third Party Property						
σ_3	1.320	0.068	1.309	0.066	1.349	0.068
α_{13}	0.677	0.088	0.615	0.080	0.617	0.079
α_{23}	1.383	0.253	1.528	0.271	1.324	0.217
intercept	1.071	0.134	1.035	0.132	0.841	0.120
vehicle age 1-10	-0.008	0.098	-0.054	0.094	-0.036	0.092
vehicle age $\gg 11$	-0.022	0.198	0.030	0.194	0.078	0.193
year	0.031	0.007	0.043	0.007	0.046	0.007
Copula						
ρ_{12}	-	-	0.250	0.049	0.241	0.054
ρ_{13}	-	-	0.163	0.063	0.169	0.074
ρ_{23}	-	-	0.310	0.017	0.330	0.019
ν	-	-	-	-	6.013	0.688

Individual risk rating

- One application considered in the paper: individual risk rating.
- The estimated model allowed us to calculate predictive means for several alternative policy designs.
  - based on the 2001 portfolio of the insurer of $n = 13,739$ policies.
- For alternative designs, we considered four random variables:
  - individuals losses, y_{ijk}
  - the sum of losses from a type, $S_{i,k} = y_{i,1,k} + \ldots + y_{i,N_i,k}$
  - the sum of losses from a specific event, $S_{EVENT,i,j} = y_{i,j,1} + y_{i,j,2} + y_{i,j,3}$, and
  - an overall loss per policy, $S_i = S_{i,1} + S_{i,2} + S_{i,3} = S_{EVENT,i,1} + ... + S_{EVENT,i,N_i}$.
- These random variables are some ways of "unbundling" the coverage, quite similar to decomposing a financial contract into primitive components for risk analysis.

Modifications to standard coverage

- We can also analyze modifications to standard coverage such as:
    - deductibles d
    - coverage limits u
    - coinsurance percentages α
- The presence of any of these modifications alters the loss function:

$$g(y;\alpha,d,u)=\begin{cases} 0 & y<d \\ \alpha(y-d) & d\le y<u \\ \alpha(u-d) & y\ge u \end{cases}.$$

Calculating the predictive means

- Define $\mu_{ik} = \mathrm{E}(y_{ijk} | N_i, K_i = k)$ from the conditional severity model with an analytic expression

$$\mu_{ik} = \exp(\mathbf{x}_{ik}^{'}\beta_k)\frac{\mathrm{B}(\alpha_{1k} + \sigma_k, \alpha_{2k} - \sigma_k)}{\mathrm{B}(\alpha_{1k}, \alpha_{1k})}.$$

- In the presence of policy modifications, we approximate this using simulation (Appendix A.2).
- Basic probability calculations show that:

$$\mathrm{E}(y_{ijk}) = \Pr(N_i = 1)\Pr(K_i = k)\mu_{ik},$$

$$\mathrm{E}(S_{i,k}) = \mu_{ik}\Pr(K_i = k)\sum_{n=1}^{\infty} n\Pr(N_i = n),$$

$$\mathrm{E}(S_{EVENT,i,j}) = \Pr(N_i = 1)\sum_{k=1}^{3}\mu_{ik}\Pr(K_i = k), \text{and}$$

$$\mathrm{E}(S_i) = \mathrm{E}(S_{i,1}) + \mathrm{E}(S_{i,2}) + \mathrm{E}(S_{i,3}).$$

A case study

- To illustrate the calculations, we looked at a randomly selected policyholder from our database with characteristic:
  - 50-year old female driver who owns a Toyota Corolla manufactured in year 2000 with a 1332 cubic inch capacity.
  - for losses based on a coverage type, we chose “own damage” because the risk factors NCD and age turned out to be statistically significant for this coverage type.
- The point of this exercise is to evaluate and compare the financial significance.

Predictive means by level of NCD and by insured's age

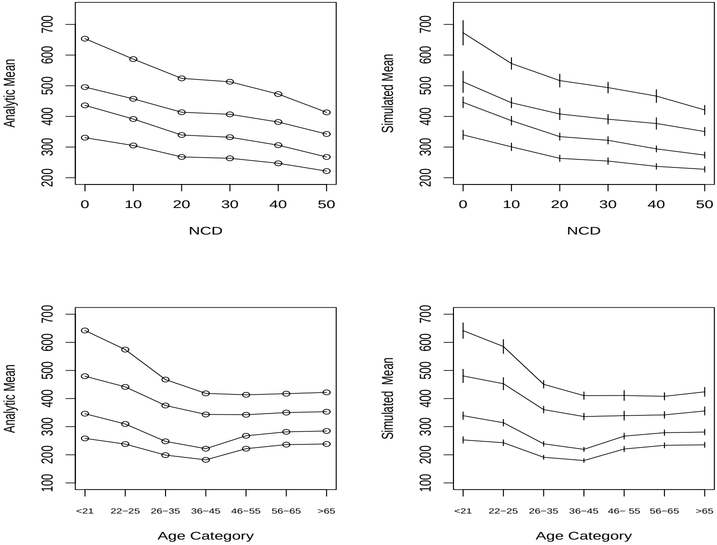
Table 3. Predictive Mean by Level of NCD

Type of Random Variable	Level of NCD					
	0	10	20	30	40	50
Individual Loss (Own Damage)	330.67	305.07	267.86	263.44	247.15	221.76
Sum of Losses from a Type (Own Damage)	436.09	391.53	339.33	332.11	306.18	267.63
Sum of Losses from a Specific Event	495.63	457.25	413.68	406.85	381.70	342.48
Overall Loss per Policy	653.63	586.85	524.05	512.90	472.86	413.31

Table 4. Predictive Mean by Insured's Age

Type of Random Variable	Insured's Age						
	≤ 21	22-25	26-35	36-45	46-55	56-65	≥ 66
Individual Loss (Own Damage)	258.41	238.03	198.87	182.04	221.76	236.23	238.33
Sum of Losses from a Type (Own Damage)	346.08	309.48	247.67	221.72	267.63	281.59	284.62
Sum of Losses from a Specific Event	479.46	441.66	375.35	343.59	342.48	350.20	353.31
Overall Loss per Policy	642.14	574.24	467.45	418.47	413.31	417.44	421.93

Predictive means and confidence intervals



Driven by frequency or severity?

Table A.5. Effect of NCD on Analytic Predictive Mean

NCD	0	10	20	30	40	50
	Probability of no accident under various NCD					
No accident	0.916	0.924	0.928	0.929	0.935	0.942
	Expected losses under various NCD					
Third party injury	10.669	10.669	10.669	10.669	10.669	10.669
Own damage	2.532	2.532	2.320	2.320	2.320	2.320
Third party property	2.765	2.765	2.765	2.765	2.765	2.765

Table A.6. Effect of Age Category on Analytic Predictive Mean

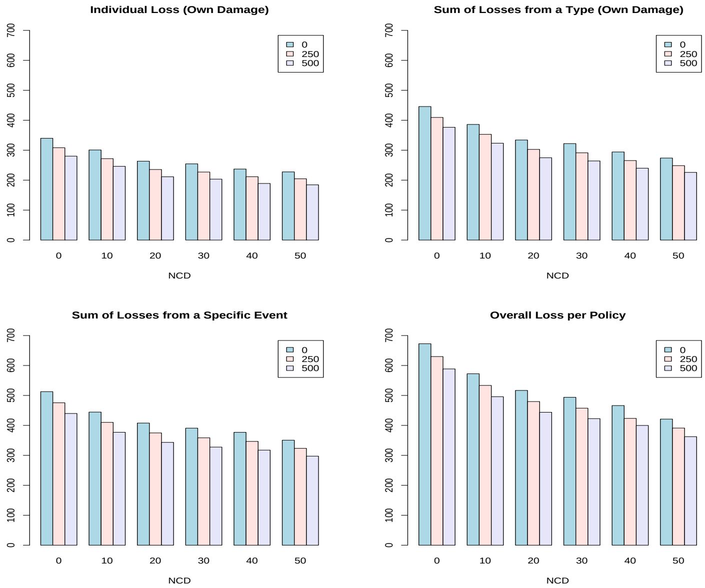
Age	≤ 21	22-25	26-35	36-45	46-55	56-65	≥ 66
	Probability of no accident under various age category						
No accident	0.912	0.920	0.933	0.940	0.942	0.946	0.945
	Probability of losses type under various age category						
Third party injury	0.027	0.027	0.027	0.027	0.031	0.031	0.031
Own damage	0.686	0.686	0.686	0.686	0.870	0.870	0.870
Third party property	0.408	0.408	0.408	0.408	0.277	0.277	0.277
	Expected losses under various age category						
Third party injury	10.669	10.669	10.669	10.669	10.669	10.669	10.669
Own damage	2.407	2.407	2.320	2.320	2.320	2.618	2.618
Third party property	2.765	2.765	2.765	2.765	2.765	2.765	2.765

Coverage modifications by level of NCD

Table 5. Simulated Predictive Mean by Level of NCD and Coverage Modifications

Coverage Modification			Level of NCD					
Deductible	Limits	Coinsurance	0	10	20	30	40	50
Individual Loss (Own Damage)								
0	none	1	339.78	300.78	263.28	254.40	237.10	227.57
250	none	1	308.24	271.72	235.53	227.11	211.45	204.54
500	none	1	280.19	246.14	211.32	203.43	188.94	184.39
0	25,000	1	331.55	295.08	260.77	250.53	235.42	225.03
0	50,000	1	337.00	300.00	263.28	254.36	237.10	227.27
0	none	0.75	254.84	225.59	197.46	190.80	177.82	170.68
0	none	0.5	169.89	150.39	131.64	127.20	118.55	113.78
250	25,000	0.75	225.00	199.51	174.76	167.43	157.33	151.50
500	50,000	0.75	208.05	184.02	158.49	152.54	141.70	138.07
Sum of Losses from a Type (Own Damage)								
0	none	1	445.81	386.04	334.05	322.09	294.09	273.82
250	none	1	409.38	352.94	302.65	291.29	265.41	248.43
500	none	1	376.47	323.36	274.82	264.12	239.90	225.93
0	25,000	1	434.86	378.55	330.50	316.57	291.78	270.39
0	50,000	1	442.35	385.05	333.98	321.87	294.07	273.40
0	none	0.75	334.36	289.53	250.54	241.56	220.56	205.37
0	none	0.5	222.91	193.02	167.03	161.04	147.04	136.91
250	25,000	0.75	298.82	259.09	224.32	214.33	197.33	183.75
500	50,000	0.75	279.75	241.77	206.06	197.94	179.91	169.13
Sum of Losses from a Specific Event								
0	none	1	512.74	444.50	407.84	390.87	376.92	350.65
250	none	1	475.56	410.12	374.90	358.54	346.58	323.41
500	none	1	439.84	377.11	343.33	327.64	317.47	297.37
0	25,000	1	483.88	433.28	394.80	380.54	359.31	340.67
0	50,000	1	494.20	442.06	401.99	388.21	367.02	348.79
0	none	0.75	384.55	333.38	305.88	293.15	282.69	262.98
0	none	0.5	256.37	222.25	203.92	195.44	188.46	175.32
250	25,000	0.75	335.02	299.17	271.39	261.15	246.73	235.08
500	50,000	0.75	315.98	281.00	253.11	243.74	230.68	221.64
Overall Loss per Policy								
0	none	1	672.68	572.51	516.77	493.93	466.26	421.10
250	none	1	629.88	533.50	479.64	457.56	432.43	391.14
500	none	1	588.55	495.85	443.87	422.63	399.85	362.37
0	25,000	1	634.81	555.90	499.72	479.90	445.04	408.81
0	50,000	1	649.67	568.30	509.52	490.46	454.84	418.92
0	none	0.75	504.51	429.39	387.58	370.45	349.69	315.82
0	none	0.5	336.34	286.26	258.39	246.96	233.13	210.55
250	25,000	0.75	444.01	387.67	346.94	332.65	308.41	284.14
500	50,000	0.75	424.16	368.72	327.46	314.37	291.32	270.15

The effect of deductible, by NCD

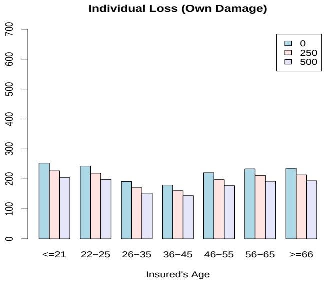


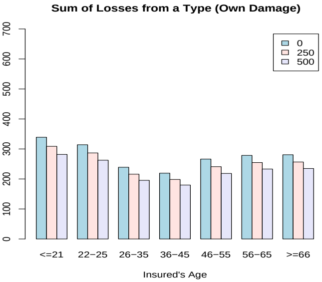
Coverage modifications by insured's age

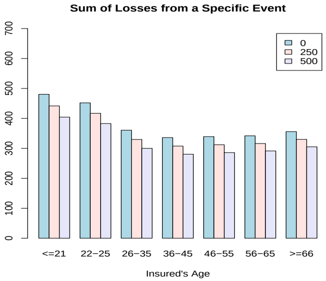
Table 6. Simulated Predictive Mean by Insured's Age and Coverage Modifications

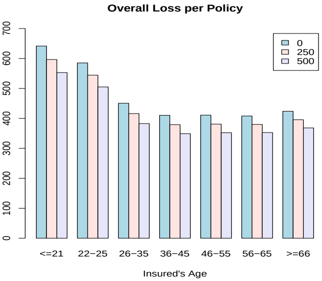
Coverage Modification			Level of Insured's Age						
Deductible	Limits	Coinsurance	$\leq$21	22-25	26-35	36-45	46-55	56-65	$\geq$66
Individual Losses (Own Damage)									
0	none	1	252.87	242.94	191.13	179.52	220.59	233.58	235.44
250	none	1	226.93	219.16	170.54	160.61	197.57	211.76	213.42
500	none	1	204.13	198.39	152.52	144.00	177.44	192.24	193.78
0	25,000	1	246.94	238.24	189.64	178.33	217.14	230.52	232.35
0	50,000	1	250.64	242.62	191.13	179.46	219.32	233.38	235.44
0	none	0.75	189.65	182.21	143.35	134.64	165.44	175.19	176.58
0	none	0.5	126.43	121.47	95.57	89.76	110.29	116.79	117.72
250	25,000	0.75	165.75	160.84	126.79	119.57	145.60	156.52	157.75
500	50,000	0.75	151.42	148.56	114.39	107.95	132.12	144.03	145.34
Sum of Losses from a Type (Own Damage)									
0	none	1	339.05	314.08	239.04	219.34	266.34	278.61	280.74
250	none	1	308.86	286.80	215.95	198.39	240.96	254.71	256.59
500	none	1	281.82	262.57	195.44	179.74	218.47	233.12	234.84
0	25,000	1	331.01	307.77	236.54	217.53	262.13	274.59	276.51
0	50,000	1	336.33	313.60	238.89	219.16	264.92	278.29	280.67
0	none	0.75	254.29	235.56	179.28	164.50	199.75	208.96	210.55
0	none	0.5	169.53	157.04	119.52	109.67	133.17	139.31	140.37
250	25,000	0.75	225.61	210.37	160.08	147.43	177.56	188.02	189.27
500	50,000	0.75	209.33	196.57	146.47	134.67	162.79	174.60	176.08
Sum of Losses from a specific Event									
0	none	1	480.49	452.84	360.72	336.00	339.24	341.88	355.91
250	none	1	441.68	417.13	329.75	307.68	312.02	316.15	329.97
500	none	1	404.35	382.86	300.06	280.46	285.91	291.37	305.06
0	25,000	1	461.26	434.27	356.68	329.88	326.36	335.92	341.76
0	50,000	1	471.44	444.84	360.30	333.98	331.88	341.66	351.95
0	none	0.75	360.37	339.63	270.54	252.00	254.43	256.41	266.93
0	none	0.5	240.24	226.42	180.36	168.00	169.62	170.94	177.95
250	25,000	0.75	316.83	298.92	244.28	226.17	224.35	232.65	236.87
500	50,000	0.75	296.48	281.14	224.73	208.83	208.91	218.37	225.83
Overall Loss per Policy									
0	none	1	641.63	585.21	450.69	410.37	410.93	408.05	423.90
250	none	1	596.61	544.40	416.07	379.07	380.98	379.93	395.52
500	none	1	553.07	505.04	382.74	348.87	352.15	352.76	368.17
0	25,000	1	616.34	561.58	444.58	402.51	394.26	399.93	406.63
0	50,000	1	630.29	575.81	449.98	407.74	401.61	407.27	419.34
0	none	0.75	481.22	438.91	338.02	307.78	308.20	306.04	317.92
0	none	0.5	320.82	292.60	225.34	205.19	205.46	204.03	211.95
250	25,000	0.75	428.49	390.58	307.48	278.41	273.23	278.86	283.69
500	50,000	0.75	406.30	371.73	286.52	259.68	257.13	263.98	272.71

The effect of deductible, by insured's age









Other applications done in the paper

- Evaluating the predictive distribution of a portfolio of policies.
- Examining effects of several reinsurance treaties: quota share and excess-of-loss arrangements.
- Analyzing the effects of retention limits both at the policy and portfolio level.
- We leave this out for the purpose of this talk - since additional time is needed to appreciate the details.