MTH 235 Ordinary Differential Equations

Worksheet, October 11, 2017

1. Consider the following ODE.
$$y' = -3(y-10)(y-5)(y-2)y.$$
   (a) Find the equilibrium solution(s) and draw a phase diagram of the ODE, clearly indicating the intervals where the population is increasing and where it is decreasing.
   (b) Determine the stability of the equilibria.
   (c) Make rough sketches of representative solution curves with various initial conditions (one for each interval of increase/decrease).
   (d) For the solution to the IVP $y' = -3(y-10)(y-5)(y-2)y$, $y(0) = 4$, find $\lim_{t\to\infty} y(t)$.

2. Find the general solution to the following ODE. You do not need to find an explicit form of the solution - it is okay to leave represent it as an implicit function of t.
$$y' = 2y(y-2)(y-5).$$

3. Find the solution to the following IVP.
$$y' = -\cot(t)y + 2t, \quad t \in \left[\frac{\pi}{2}, \pi\right) \qquad y\left(\frac{\pi}{2}\right) = -1.$$
   Ans.
$$y(t) = \frac{-3}{\sin(t)} - 2t\cot(t) + 2.$$

4. Use the Method of Undetermined Coefficients to find the general solution of
$$y'' - 6y' + 8y = 3e^{2t}.$$
   Ans.
$$y(t) = c_1 e^{4t} + \left(c_2 - \frac{3}{2}t\right)e^{2t}.$$

5. Use the Method of Variation of Parameters to find the general solution of
$$y'' + 9y = -2\csc(3t).$$
   Ans.
$$y(t) = c_1\cos(3t) + c_2\sin(3t) + \frac{2}{3}t\cos(3t) - \frac{2}{9}\ln(|\sin(3t)|)\sin(3t).$$

6. Consider the initial value problem
$$y'' - 4y' + 29y = 1, \quad y(0) = 0, \quad y'(0) = 1.$$
   Use Laplace Transform to find the solution.
   Ans.
$$y(t) = \frac{1}{145}\left(31e^{2t}\sin(5t) - 5e^{2t}\cos(5t) + 5\right)$$

7. Consider the initial value problem

$$y'' - 5y' + 4y = -5u(t-9), \quad y(0) = 0, \quad y'(0) = 0.$$

Use Laplace Transform to find the solution.

Ans.

$$y(t) = -5u(t-9)\left(\frac{1}{4} + \frac{e^{4(t-9)}}{12} - \frac{e^{t-9}}{3}\right)$$

8. Consider the initial value problem

$$y'' - 7y' + 6y = e^{5t}\delta(t-5), \quad y(0) = 0, \quad y'(0) = 0.$$

Use Laplace Transform to find the solution.

Ans.

$$y(t) = \frac{e^{25}}{5}u(t-5)(e^{6(t-5)} - e^{t-5})$$