Math 421-001 # Homework Assignment #14 **Due April 27, 2015**

Name ________________________________

Problem 1. Let $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\mathbf{f}(x, y) = (x^2 + |y|, y^2)$.

(a) Is $\mathbf{f}$ differentiable at $\mathbf{a} = (0, 1)$? If so, find $D\mathbf{f}(0, 1)$. Justify your answer by citing the appropriate theorems.

(b) Is $\mathbf{f}$ differentiable at $\mathbf{a} = (0, 0)$? If so, find $D\mathbf{f}(0, 0)$. Justify your answer by citing the appropriate theorems.

Problem 2. Let $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $\mathbf{f}(x, y) = (x^2 + y|y|, y^2)$.

Is $\mathbf{f}$ differentiable at $\mathbf{a} = (0, 0)$? If so, find $D\mathbf{f}(0, 0)$. Justify your answer by citing the appropriate theorems.

Problem 3. Prove that

$$f(x, y) = \begin{cases} (x^2 + y^2)\cos\dfrac{1}{\sqrt{x^2 + y^2}} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

is differentiable on $\mathbb{R}^2$, but not continuously differentiable at $(0, 0)$.

Hint: Consider Example 11.18 in the book.

Problem 3. (11.2.8) Suppose that $\mathbf{T} \in \mathcal{L}(\mathbb{R}^n\mathbb{R}^m)$. Prove that $\mathbf{T}$ is differentiable everywhere on $\mathbb{R}^n$ with $D\mathbf{T}(\mathbf{a}) = \mathbf{T}$ for all $\mathbf{a} \in \mathbb{R}^n$.