Math 421-001 **Review for Exam 2** **Spring, 2015**

I. Be able to state important definitions and theorems.

II. Review homework problems.

III. Review quizzes.

IV. Be able to prove short and straightforward theorems.

The following I consider to be *easy* to prove.

- Remark 8.21
- Remark 8.22
- Theorem 8.32
- Theorem 9.2
- Theorem 9.6

The following proofs I consider to be of *medium* difficulty for a test.

- Theorem 8.36
- Theorem 9.4 - various parts
- Theorem 9.7
- Theorem 9.26 - one of the directions
- Theorem 9.32

These are problems, which are more challenging than problems for a 50 minute exam, but will help you review the material.

1. Determine if the following statement is true or false. If true, prove it. If false, provide a counter-example.

   For every set $E \subseteq R^n$ the complement of the interior of the complement of E is the closure of E, that is $((E^c)^\circ)^c = \overline{E}$.

2. Suppose that $E_1 \supseteq E_2 \supseteq E_3 \ldots$ is a decreasing sequence of nonempty subsets of $\mathbb{R}^n$, such that for each $k \in \mathbb{N}$, E_k is a compact set. Prove that $\bigcap_{k=1}^{\infty} E_k \neq \emptyset$.