Math 421-001 **Review for Exam 1** **Spring, 2015**

I. Be able to state important definitions and theorems.

II. Review homework problems.

III. Review quizzes.

IV. Be able to prove short and straightforward theorems (e.g. problems 1 and 4).

In addition, these problems might help you review the material.

1. If P is any partition of $[a, b]$ and $Q = P \cup \{c\}$, prove that $U(f, Q) \leq U(f, P)$.

2. Prove that the function defined by

$$f(x) = \begin{cases} 2, & x \in [0,5] \cap \mathbb{Q}, \\ 4, & x \in [0,5] \setminus \mathbb{Q}. \end{cases}$$

   is not Riemann integrable on $[0, 5]$.

3. Prove that the function defined by

$$g(x) = \begin{cases} 1, & x \in [1,2), \\ 5, & x \in [2,3), \\ 7, & x = 3, \\ -5, & x \in (3,8]. \end{cases}$$

   is Riemann integrable on $[1, 8]$.

4. Use the Fundamental Theorem of Calculus to prove that if f and g are differentiable on $[a, b]$ and f' and g' are integrable on $[a, b]$, then

$$\int_a^b f'(x)g(x)\,dx = f(b)g(b) - f(a)g(a) - \int_a^b f(x)g'(x)\,dx.$$

5. Suppose f is integrable on $[0.5, 2]$ and that

$$\int_{0.5}^1 x^k f(x)\,dx = \int_1^2 x^k f(x)\,dx + 2k^2 = 3 + k^2$$

   for $k = 0, 1, 2$. Compute $\displaystyle\int_0^1 x^3 f(x^2 + 1)\,dx$.

6. Let $\mathbf{T} \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$ be defied by $\mathbf{T}(x, y) = (3y, 2x)$. Find $||\mathbf{T}||_2$. Recall that $||\mathbf{T}||_2 = \sup\limits_{||\mathbf{x}||=1} \dfrac{||\mathbf{T}(\mathbf{x})||_2}{||\mathbf{x}||_2}$.