Math 320-001 | **Review for Exam 2** | **Spring, 2016**

This exam focuses on Sections 14, 15, 17, 18, 19, 20.

I. Be able to state important definitions and theorems.

II. Review homework problems.

III. Review quizzes.

IV. Be able to prove short and straightforward theorems.

Important definitions you should be able to state

1. Convergence of an infinite series
2. Continuity of a function (sequential and $\varepsilon - \delta$ property)
3. Uniform continuity
4. Limit of a function as x tends to a along a set S (Def. 20.1)
5. Two-sided limit, left hand limit, right hand limit ($\lim_{x\to a} f(x) = L$, $\lim_{x\to a^+} f(x) = L$, $\lim_{x\to a^-} f(x) = L$)
6. Define each of the following limits in terms of quantified implications ($\varepsilon - \delta$ definition, or $M - \delta$, etc.): $\lim_{x\to a^-} f(x) = +\infty$, $\lim_{x\to -\infty} f(x) = +\infty$, ...

Important theorems you should be able to state/prove

1. State and apply: Comparison, Ratio, Root, Alternating Series, Integral Tests
2. Prove basic properties on how to work with convergent infinite series - add, multiply by a scalar, etc. (See Problem 14.5)
3. Prove: Th. 17. 2 and 20.6 - note the differences and similarities in the proofs.
4. Prove: continuity of a function at a given point using the definition and the $\varepsilon - \delta$ properties (See Examples 1, 2, 3 in Sect. 17 and related HW.)
5. Prove: properties about operations on continuous functions (See Th. 17.4, 17.5)
6. State and apply: Extreme Value Theorem (Th. 18.1), Intermediate Value Theorem (Th. 18.2)
7. State and apply: Fundamental theorems on uniform continuity (Th. 19.2, Th. 19.4, Th. 19.5)
8. Prove: properties of limits of functions (See Th. 20.4, Th. 20.5)

In addition, these problems might help you review the material.

1. Determine which of the following series converge. Justify your answer.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^3}{5^n}$

(b) $\sum_{n=1}^{\infty} \frac{n-5}{2n}$

(c) $\sum_{n=1}^{\infty} \frac{n-5}{2n^2}$

(d) $\sum_{n=1}^{\infty} \frac{n+5}{2n^3}$

(e) $\sum_{n=0}^{\infty} \frac{3}{(-2)^n}$ - calculate the sum.

(f) $\sum_{n=1}^{\infty} \frac{3(-1)^n}{n}$

(g) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2}$

2. Prove that the function $\sqrt{4x+1}$ is continuous at $x_0 = 2$ by verifying the $\varepsilon - \delta$ property.

3. Prove that the function $\sqrt{4x+1}$ is uniformly continuous on $[2, \infty$ by directly verifying the $\varepsilon - \delta$ property.

4. Prove that the function $f(x) = ((2x-1)^6 + \sqrt{3x^2+16})^5$ is uniformly continuous on $(0, 10)$ using the appropriate theorems.

5. Complete the following statements:

   (a) $\lim_{x \to a^-} f(x) = +\infty$ if and only if for any ________________ there exits ________________ such that ________________ implies ________________.

   (b) $\lim_{x \to a} f(x) = L$ if and only if for any ________________ there exits ________________ such that ________________ implies ________________.

   (c) $\lim_{x \to -\infty} f(x) = L$ if and only if for any ________________ there exits ________________ such that ________________ implies ________________.

6. Prove, using the above problem.

   (a) $\lim_{x \to 9^-} \frac{7}{9-x} = +\infty$.

   (b) $\lim_{x \to 10} \frac{7}{9-x} = -7$. (How could we prove this in a simpler way than using the above problem?)

   (c) $\lim_{x \to \infty} \frac{7x}{9-x} = -7$