Math 320-001 | **Review for Exam 1** | **Spring, 2016**

I. Be able to state important definitions and theorems.

II. Review homework problems.

III. Review quizzes.

IV. Be able to prove short and straightforward theorems (e.g. problems 1 and 4).

Important definitions you should be able to state

1. Upper bound, lower bound, supremum, infimum for a set in $\mathbb{R}$.
2. What do we mean by $\lim_{n\to\infty} s_n = L$, $\lim_{n\to\infty} s_n = +\infty$, $\lim_{n\to\infty} s_n = -\infty$?
3. Increasing, decreasing sequence.
4. Definition of limsup and liminf.
5. Cauchy sequence.

Important theorems/axioms you should be able to state/prove

1. Proving basic properties of real numbers based on given axioms.
2. Completeness Axiom (state)
3. Proving basic properties of inf and sup (see problems 4.5-4.9, 4.14, 4.16)
4. Proving a given sequence converges using the definition (see Sect. 8)
5. Prove: Convergent sequences are bounded. (see Sect. 9)
6. Prove: If (s_n) and (t_n) converge, for $a, b \in \mathbb{R}$, prove $\lim_{n\to\infty}(as_n + bt_n) = a \lim_{n\to\infty} s_n + b \lim_{n\to\infty} t_n$. (see Sect. 9)
7. Proving a given sequence converges using limit theorems (see Sect. 9)
8. Prove: Given $s_n > 0$ for all $n \in \mathbb{N}$, prove $\lim_{n\to\infty} s_n = +\infty$ if and only if $\lim_{n\to\infty} \frac{1}{s_n} = 0$. (see Sect. 9)
9. Be able to use: All bounded monotone sequences converge.
10. Prove: If a sequence is convergent, then it is Cauchy. (Sect 10)

In addition, these problems might help you review the material.

1. Let A be a nonempty subset of $\mathbb{R}$ that is bounded below, and $B = \{a + 7 : a \in A\}$. Prove that B is also bounded below and $\inf B = 7 + \inf A$.

2. Use the definition of convergence to prove $\lim_{b \to \infty} \frac{3n^3 + 7n + 1}{n^3 - n - 3} = 3$.

3. Let $a_n = 5 + 3(-1)^n$ for $n \in \mathbb{N}$. Prove (a_n) diverges.

4. Let $a_1 = 1$ and $a_{n+1} = \frac{1}{5}(3a_n + 1)$ for $n \geq 1$. Prove $\lim_{n \to \infty} a_n$ exists and find the limit.

5. Use the definition to prove that if $\lim_{n \to \infty} a_n = -\infty$ and $\lim_{n \to \infty} b_n = 7$, then $\lim_{n \to \infty} (2a_n + 3b_n) = -\infty$