Math 310 **Homework assignment, Section 1.3** **Due Friday, 9/12.**

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2. Let p be an integer other than $0, \pm 1$. Prove that p is prime if and only if for each $a \in \mathbb{Z}$ either $\gcd(a,p) = 1$ or $p \mid a$.

4. Let p be an integer other than 0, ± 1 with the following property: Whenver b and c are integers such that $p \mid bc$, then $p \mid b$ or $p \mid c$. Prove that p is prime.

Hint: If d is a divisor of p, say $p = dt$ for some $d, t \in \mathbb{Z}$, then $p \mid d$ or $p \mid t$. Show that this implies $d = \pm p$ or $d = \pm 1$.

10. Assume $a, b, c, d \in \mathbb{Z}$. Prove or disprove each of the following statements.

(a) If p is prime and $p \mid (a^2 + b^2)$ and $p \mid (c^2 + d^2)$, then $a^2 - c^2$.

(b) If p is prime and $p \mid (a^2 + b^2)$ and $p \mid (c^2 + d^2)$, then $a^2 + c^2$.

(c) If p is prime and $p \mid a$ and $p \mid (a^2 + b^2)$, then b.

25. (Bonus - you can use this problem to substitute two problems of your choice from group A assigned today and Monday, 9/8.) Prove or disprove: If n is an integer and $n > 2$, then there exists a prime p such that $n < p < n!$.