Proofs Involving Inequalities

Consider the following basic facts regarding inequalities.

A1 For all real numbers a, b, c, if $a \leq b$ and $b \leq c$ then $a \leq c$.

A2 For all real numbers a, b, c, if $a \leq b$ then $a + c \leq b + c$.

A3 For all real numbers a, b, c, if $a \leq b$ and $0 \leq c$ then $ac \leq bc$.

Prove the statements below using A1-A3, together with any basic facts about *equality* $=$.

1. For all real numbers a, if $a \leq 0$ then $0 \leq -a$.

2. For all real numbers a, b, if $b \leq a$ and $a \leq 0$, then $a^2 \leq b^2$.

Hints:

1. Use A2 with $c = -a$.

2. This is similar to the statement proven in class, except you should use Problem 1 so that A3 applies.