

Prove the following proposition.

Let $s_n \neq 0 \forall n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} s_n = s$, where $s \neq 0$, then $\exists m > 0$ such that $\inf (\{|s_n| \mid n \in \mathbb{N}\}) > m > 0$.

The following steps might be helpful.

1. Draw a picture to convince yourself that the proposition makes sense.
2. Use the fact that $\lim_{n \rightarrow \infty} s_n = s$ and take $\varepsilon = \frac{|s|}{2}$.
3. Show that $\exists N$ such that $|s_n| > \frac{|s|}{2} \forall n > N, n \in \mathbb{N}$. You can write $|s| = |s - s_n + s_n|$, and apply the Triangle Inequality.
4. How can you define a lower bound, $m > 0$, for the set $\{|s_n| \mid n \in \mathbb{N}\}$?