

1. Use the formal definition of limit of a sequence to prove the following.

(a) $\lim_{n \rightarrow \infty} \frac{2n + 1}{n + 1} = 2$

(b) $\lim_{n \rightarrow \infty} \frac{1}{3n - 1} = 0$

2. Prove the following proposition.

Let $x, y \in \mathbb{R}$. Then $x = y$ if and only if $\forall \varepsilon > 0, |x - y| < \varepsilon$.

Hint: For the direction “If $\forall \varepsilon > 0, |x - y| < \varepsilon$, then $x = y$,” prove the contrapositive statement. Take $\varepsilon = \frac{x - y}{2}$, assuming, without loss of generality, that $x > y$.