

1. Let A and B be bounded subsets of \mathbb{R} such that $A \subseteq B$. Prove that $\inf(B) \leq \inf(A)$.
2. Using only the axioms for \mathbb{R} , show that 1 is a positive real number.

Hint:

- (a) Assume, by way of contradiction, $1 \notin \mathbb{R}^{>0}$.
- (b) Using Axiom 8.26(iv) what can you conclude about -1 ?
- (c) Show that $(-1) \cdot (-1) = 1$ by starting from $0 \cdot 0 = (-1 + 1) \cdot (-1 + 1)$, and using uniqueness of the additive inverse (this is not an axiom but we have proven this in class for groups). What axiom allows you to expand the expression?
- (d) Use Axiom 8.26(ii) - what can you conclude about 1? Why is this a contradiction?
- (e) Clearly state your conclusion.