Math 299 **Essay 1** **Fall 2013**

Give a careful proof of the following, following the steps below.

PROPOSITION: Let $f : A \to B$ and $g : B \to C$ be two injective functions. Then the composite function $g \circ f : A \to C$ is also injective.

Proof Steps

1. Get a feel for the proposition by drawing a few dot-and-arrow pictures of injective f and g on small finite sets.
2. At the top of your page, state the proposition. Do not assume your reader knows what you are talking about!
3. At the beginning of the proof, write the *hypothesis*: what is given or assumed, the setup of the proposition.
4. Leaving plenty of space on your proof page, at the bottom write the *conclusion*: what is to be deduced from the hypothesis setup.
5. Review the defintion of each term in the hypothesis and conclusion. Write each definition near the term: these will almost certainly be needed in your proof.
6. The body of the proof is a bridge of logical deductions from the hypothesis to the conclusion.
7. The standard way to prove any function h is injective is to assume that $h(a_1) = h(a_2)$ for unknown input values a_1, a_2, and then deduce that $a_1 = a_2$. You can think of $h(a_1) = h(a_2)$ as a sub-hypothesis, an extra assumption which you can use in the course of the proof, eventually deducing the sub-conculsion $a_1 = a_2$.
8. Finish off by explaining why the implication (sub-hypothesis $\Rightarrow$ sub-conclusion) leads to the main conclusion of the proposition.
9. Once all this is done, do not just turn in the mess. Make a final clean draft with a clear logical flow, leaving out anything that was not needed.
10. Judge your audience: leave out details that will be obvious to them. As in other forms of expository writing, more words = more pain. We consider a proof rigorous not if it spells out every last detail, but rather if it is clear to the audience how every last detail could be supplied if wanted. Assume your audience is one of your classmates who has not done this assignment and needs a good deal of explanation. Do NOT assume the audience is the professor, who knows all about the problem and needs only a hint that you understand the proof.