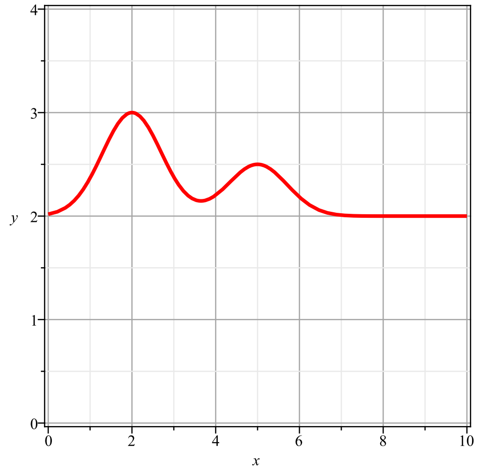
Review for Final Exam

MTH132-040, Calculus I

(1) The graph below describes the population of fruit flies (measured in hundreds) as a function of time, over a period of 10 days.



(a) Over what period of time is the instantaneous rate of change of the population negative?

(b) Over what period of time is the instantaneous rate of change of the population positive?

(c) On approximately what day is the derivative of the function, giving the population as a function of time, the greatest?

(d) Calculate the average rate of change of the population between day 2 and day 5.

(2) Calculate the following limits.

(a) $\displaystyle\lim_{x\to 3} \frac{3x^2 - 5x + \pi}{x^2 - 3}$ (b) $\displaystyle\lim_{x\to 2^-} \frac{x+5}{x^2 - 3x + 2}$ (c) $\displaystyle\lim_{h\to 0} \frac{\frac{1}{x+5+h} - \frac{1}{x+5}}{h}$

(d) $\displaystyle\lim_{x\to -3} \frac{2 - \sqrt{x^2 - 5}}{x+3} + 3x - 1$ (e) $\displaystyle\lim_{x\to\infty} \frac{5x^3 - 7x + 1}{1 - x^4}$ (f) $\displaystyle\lim_{x\to -\infty} \frac{|x+5|}{x-4}$

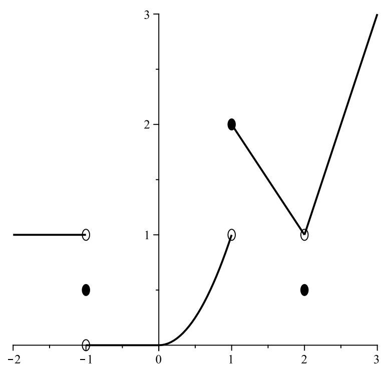
(g) $\displaystyle\lim_{t\to 0} \frac{7\tan(3t)}{5t}$

(h) Suppose $\lim_{s\to 3} f(s) = -1$ and $\lim_{s\to 3} g(s) = 6$, find $\lim_{s\to 3} \left(f^2(s) + 5f(s)g(s)\right)$.

(3) Use the Sandwich Theorem to find $\lim_{p\to 5} S(p)$, provided that $\dfrac{6-p}{p-2} \le S(p) \le \dfrac{\sin(p-5)}{3p-15}$.

(4) Using the graph of the function $y = f(x)$ given below, evaluate the following:

$$\lim_{x\to -1} f(x) = DNE$$
$$f(-1) = 0.5$$
$$\lim_{x\to 1^-} f(x) = 1$$
$$\lim_{x\to 1^+} f(x) = 2$$
$$f(1) = 2$$
$$\lim_{x\to 2} f(x) = 1$$
$$f(2) = 0.5$$



Which one of the discontinuities is removable? Why?

Give a definition of a *continuous* function.

(5) Let the function f be defined as follows.

$$f(x) = \begin{cases} 2x^2 + 5, & x < -1 \\ x + a, & x \geq -1. \end{cases}$$

Determine the value of a, which ensures that the function is continuous on its domain.

(6) Use the Intermediate Value Theorem to show that the equation $x^2 + 5 = 2^x$ has a solution. Carefully explain how you applied the theorem.

(7) Find the horizontal, vertical and oblique asymptotes of the following functions. Explain what they tell us about the behavior of the function when the independent variable is near a given point or approaches $\pm\infty$.

(a) $\dfrac{2x+6}{x^2-9} + 2$ (b) $\dfrac{x^2+3x-1}{x+1}$

(8) Find the equation of the tangent line to the graph of $y = \frac{16}{x} - 2\sqrt[3]{x} + \sqrt{5}$ at $x = 8$.

(9) Give examples of functions that are not differentiable.

(10) Can a function be differentiable but not continuous? How about continuous, but not differentiable?

(11) Find the derivative of the following functions.

(a) $f(t) = 3t^5 - \dfrac{\pi}{t^5} + \sqrt[3]{t^7}$

(b) $g(s) = (2s + \dfrac{1}{s} + 3) \cdot (3s + 7s^2 - 1) \cdot (\sqrt[3]{\pi} - 21s^3)$

(c) $p(x) = \dfrac{(2x+1)(3x-1)}{4x^2 - x + 5}$

(12) At a time t the temperature of in a Petri dish as a function of time is given by $T(t) = t^3 - 6t^2 + 9t$, where T is measured in degrees Celsius, and t is measured in hours.

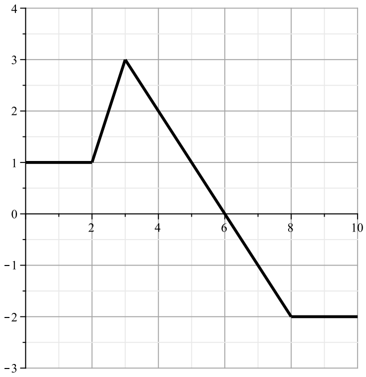
(a) Find the rate of change of temperature each time $T''(t) = 0$.
(b) What does it say about the temperature at time t if $T'(t) > 0$?
(c) What does it say about the temperature at time t if $T'(t) < 0$?
(d) What does it say about the rate of change of the temperature at time t if $T''(t) > 0$?

(13) An ant is moving along a straight line and its motion as a function of time as described in the graph below.

(a) Over what period of time is the ant moving the fastest?

(b) Over what period of time is the ant moving to the right/moving to the left/standing still?

(c) Graph the ant's velocity (where defined) as a function of time.



(14) Find the derivative of each of the following functions.

(a) $\sec(x^2+3)\sin(5x-1)$

(b) $\dfrac{\cos(\sin(x))}{x^5-3x+1}$

(c) $\tan(\cos((x^6-3x+8)^7))$

(15) Assume $f'(x) = h(x) \cdot g(x)$. Find $\dfrac{d}{dx} f(p(x))$ at $x = 1$, given that

x	$g(x)$	$h(x)$	$p(x)$	$g'(x)$	$h'(x)$	$p'(x)$
1	5	-3	7	2	-9	3
3	2	6	-1	3	0	5
7	4	-5	2	3	-1	-2

(16) Find an equation of the tangent line to the curve $x^2y^2 = 9$ at the point $(-1, 3)$.

(17) When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50 cm?

(18) Show that the linearization of $f(x) = (1+x)^k$ at $x = 0$ is $L(x) = 1 + kx$. Use this to find the linear approximation to $(1.02)^8$.

(19) State the *Extreme Value Theorem.* Give examples of functions which violate the theorem's assumptions and conclusion.

(20) For a continuous function f, defined on a closed interval $[a, b]$, where can the absolute extrema of the function occur?

(21) Find the absolute minimum and absolute maximum values of the following functions. Identify the intervals of increase and decrease.

(a) $f(x) = \sqrt{4-x^2}$, $\quad -2 \leq x \leq 1$

(b) $g(t) = t^3 - 3t^2$, $\quad x \in [-5, 0]$

(22) State *Rolle's Theorem.* Provide a graph to explain the theorem.

(23) State the *Mean Value Theorem.* Provide a graph to explain the theorem.

(24) Assume f is defined on the whole real line and let $f'(x) = \dfrac{(x-3)^2(x+1)}{\sqrt[3]{x+2}}$, $x \neq -2$. What are the critical points of f? On what intervals is f increasing or decreasing? At what points, if any, does f assume local minimum or local maximum values?

(25) Assume f is defined on the whole real line and let $f'(x) = (8x - 5x^2)(4 - x)^2$. What are the critical points of f? On what intervals is f increasing or decreasing? At what points, if any, does f assume local minimum or local maximum values? Where is the function concave up/ concave down? Sketch the graph of the function.

(26) Let $y = \dfrac{2x^2 + x - 1}{x^2 - 1}$.
  (a) Find any vertical and horizontal asymptotes, as well as removable singularities, if any.
  (b) Find the intervals of increase/decrease.
  (c) Find the local extrema, if any.
  (d) Find the intervals where the function is concave up/down.
  (e) Find the inflection points, if any.
  (f) Find the y-intercept.
  (g) Sketch a possible graph of the function.

(27) Sketch the graph of a function f which satisfies the following conditions.
  (a) f is twice differentiable for all x, except $x = 3$
  (b) $f'(3)$ is not defined
  (c) $f'(x) > 0$ on $(3, \infty)$
  (d) $f'(x) < 0$ on $(-\infty, 3)$
  (e) $f''(x) < 0$ for all x, except $x = 3$

(28) Jane is 2 miles offshore in a boat and wishes to reach a coastal village 6 miles down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time?

(29) You are planning to make an open top rectangular box from an 8 inch by 15 inch piece of cardboard by cutting congruent squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you can make this way, and what is the volume?

(30) Explain why Newton's method fails to find the root of the function $f(x) = \sqrt{x}$, no matter what initial guess we choose.

(31) Give a definition of an antiderivative of a function $f(x)$. Is it unique?

(32) Explain the difference between a *general* and a *particular* solution of a differential equation. Draw the general solution to $y' = 2x$ and the particular solution which satisfies $y(0) = -3$.

(33) Solve the following initial value problems.
  (a) $\dfrac{dy}{dt} = \dfrac{1}{x^2} + x, \quad x > 0, \quad y(2) = 1$
  (b) $\dfrac{d^2r}{dt^2} = \dfrac{2}{t^3}, \quad \left.\dfrac{dr}{dt}\right|_{t=1} = 1, \quad r(1) = 1$

(34) You are driving along a highway at a steady 60 mph (88 ft/sec) when you see an accident ahead and slam on the breaks. What constant deceleration is required to stop your car in 242 ft?

(35) Assume the rate of change in number of bacteria on day t, $B'(t)$, is given by the following table.

Day	0	1	2	3	4
$B'(t)$	1	3	1	0	5

(a) Give a lower estimate of the total change in population of bacteria over the 4 days.
(b) Give an upper estimate of the total change in population of bacteria over the 4 days.
(c) Give an estimate of the total change in population of bacteria over the 4 days using 4 subintervals of length 1 with left-endpoint values.
(d) What is an upper estimate of the total population of bacteria on day 4, if there were 100 bacteria on day 1?

(36) Find the average temperature for Monday, if the temperature for that day as a function of time t was given by

$$T(t) = 40 + 10\sin(t), \qquad 0 \leq t \leq 24.$$

(37) Evaluate the following limit.

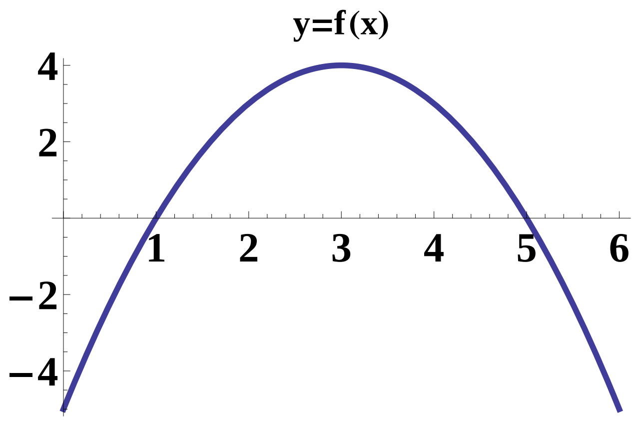
$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n^2}(5+3k)$$

(38) Express the following sum in sigma notation.

$$\frac{3}{2} - \frac{5}{4} + \frac{7}{8} - \frac{9}{16} + \frac{11}{32} - \frac{13}{64} + \frac{15}{128} - \frac{17}{256}$$

(39) Assume $\int_{-1}^{3} f(x)\,dx = 7$, $\int_{2}^{3} f(x)\,dx = 10$, $\int_{-1}^{2} g(x)\,dx = 4$. Find $\int_{-1}^{2} (3f(x) - 2g(x))\,dx$.
Given this information, can one find $\int_{-1}^{2} f(x)^2\,dx$? How about $\left(\int_{-1}^{2} f(x)\,dx\right)^2$?

(40) Let $F(x)$ be defined on the interval $[0,6]$ by $F(x) = \int_0^x f(t)\,dt$, where the graph of $y = f(x)$ is given in the figure below. Use the figure to determine the intervals of increase and decrease



of $F(x)$. Can you determine if $F(1)$ is positive or negative? How about $F(4)$? Explain your reasoning.

(41) Find the derivative of each of the following functions.

(a) $f(s) = \int_{-5}^{3s} \left(5\sin^2(3x) + 7^x\right)\, dx$

(b) $g(u) = \int_{-\sin(u)}^{\cos(u)} \left(e^{t^2} - 3t + 9\right)\, dt$

(42) Evaluate the following indefinite integrals

(a) $\int \left((\sqrt{x} + 3x^2)(x+2)\right)\, dx$

(b) $\int \left(\sin(5x) + \cos(x/3) + \pi - \sec^2(3x)\right)\, dx$

(c) $\int \sin^5 \frac{x}{3} \cos \frac{x}{3}\, dt$

(d) $\int \frac{1}{x^2}\sqrt{2 - \frac{1}{x}}\, dx$

(e) $\int s^3\sqrt{s^2+1}\, ds$

(43) Evaluate the following definite integrals

(a) $\int_{-1}^{1} (3x^2 - 4x + 7)\, dx$

(b) $\int_0^1 \frac{36}{(2x+1)^3}\, dx$

(c) $\int_0^{\pi/2} \frac{3\sin x \cos x}{\sqrt{1 + 3\sin^2 x}}\, dx$

(44) Find the area enclosed between $y = x^2$ and $y = -x^2 + 4x$.

(45) Find the area enclosed between $x = 12y^2 - 12y^3$ and $x = y^2 - 2y$.