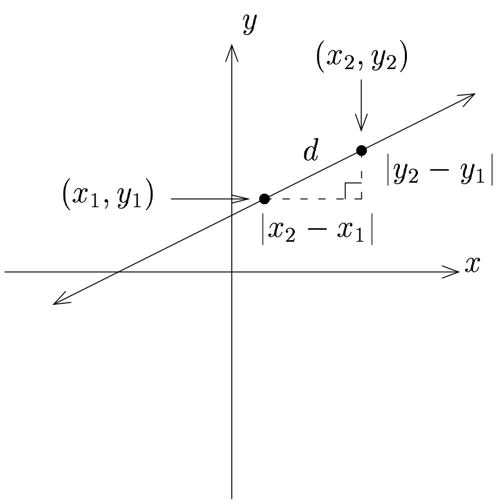
9.1 Distance Formula and Circles

A. Distance Formula

We seek a formula for the distance between two points:



By the Pythagorean Theorem, $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Since distance is positive, we have:

Distance Formula: $\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$

B. Example

Find the distance between $(-1, 2)$ and $(3, -4)$

Solution

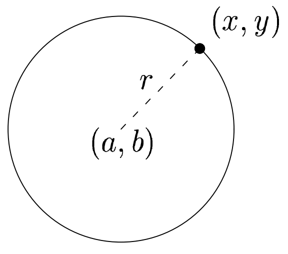
Use the distance formula:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-1))^2 + (-4 - 2)^2} \\ &= \sqrt{(4)^2 + (-6)^2} \\ &= \sqrt{16 + 36} \\ &= \sqrt{52} \end{aligned}$$

Ans $\boxed{2\sqrt{13}}$

C. Circles

A **circle** is the set of points a fixed distance r from a center (a, b):



By the distance formula, $r = \sqrt{(x - a)^2 + (y - b)^2}$

Eliminating the radical, we get:

Equation of Circle in Standard Form: $\boxed{(x-a)^2+(y-b)^2=r^2}$

Note: r is called the **radius** of the circle.

D. Examples

Example 1: Find the equation of a circle with center $(2,-1)$ and radius 4.

Solution

The equation of a circle in standard form: $(x-a)^2+(y-b)^2=r^2$

Thus, we have: $(x-2)^2+(y+1)^2=4^2$

Ans $\boxed{(x-2)^2+(y+1)^2=16}$

Example 2: Given a circle $x^2+(y-3)^2=5$, find the center and radius.

Solution

Since the equation of a circle in standard form is $(x-a)^2+(y-b)^2=r^2$, we have

Ans

center:	$(0,3)$
radius:	$\sqrt{5}$

E. Putting the Equation of a Circle in Standard Form

Sometimes the equation of a circle is **not** in standard form.
To put it in standard form, we complete the square in both x and y.
In standard form, it is easy to identify the center and radius of the circle.

Example 1: Put the equation of the circle $x^2 + y^2 - 6x + 2y = 15$ into standard form.

Solution

$$x^2 + y^2 - 6x + 2y = 15$$

$$(x^2 - 6x) + (y^2 + 2y) = 15$$

$$[(x^2 - 6x + 9) - 9] + [y^2 + 2y + 1] - 1 = 15$$

$$(x - 3)^2 - 9 + (y + 1)^2 - 1 = 15$$

Ans $\boxed{(x - 3)^2 + (y + 1)^2 = 25}$

Example 2: Put the equation of the circle $2x^2 + 2y^2 - 10x - 12y = 7$ into standard form.

Solution

$$2x^2 + 2y^2 - 10x - 12y = 7$$

$$(2x^2 - 10x) + (2y^2 - 12y) = 7$$

$$2(x^2 - 5x) + 2(y^2 - 6y) = 7$$

$$\left[2\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{2}\right] + \left[2(y^2 - 6y + 9) - 18\right] = 7$$

$$\left[2\left(x-\frac{5}{2}\right)^2-\frac{25}{2}\right]+\left[2(y-3)^2-18\right]=7$$

$$2\left(x-\frac{5}{2}\right)^2+2(y-3)^2-\frac{25}{2}-\frac{36}{2}=7$$

$$2\left(x-\frac{5}{2}\right)^2+2(y-3)^2-\frac{61}{2}=7$$

$$2\left(x-\frac{5}{2}\right)^2+2(y-3)^2=7+\frac{61}{2}$$

$$2\left(x-\frac{5}{2}\right)^2+2(y-3)^2=\frac{14}{2}+\frac{61}{2}$$

$$2\left(x-\frac{5}{2}\right)^2+2(y-3)^2=\frac{75}{2}$$

Ans $\boxed{\left(x-\frac{5}{2}\right)^2+(y-3)^2=\frac{75}{4}}$

F. Graphing Circles

To graph a circle:

1. Put the equation in standard form.
2. Find the center and radius.
3. Find the x and y intercepts.
4. Plot the x and y intercepts.
   Going any direction from the center by a radius amount reaches the circle.
5. Connect the dots.

Example: Find the center, radius, x and y intercepts of the circle, where $x^2 + y^2 - 2x + 8y = -5$. Then graph the circle.

Solution

First, put the circle in standard form:

$$x^2 + y^2 - 2x + 8y = -5$$

$$(x^2 - 2x) + (y^2 + 8y) = -5$$

$$(x^2 - 2x + 1) - 1 + (y^2 + 8y + 16) - 16 = -5$$

$$(x - 1)^2 + (y + 4)^2 - 17 = -5$$

$$(x - 1)^2 + (y + 4)^2 = 12$$

center: $\boxed{(1, -4)}$

radius: $\sqrt{12} = \boxed{2\sqrt{3}}$

x**-intercepts:** set $y = 0$:

$$\begin{aligned} (x-1)^2 + (0+4)^2 &= 12 \\ (x-1)^2 + 16 &= 12 \\ (x-1)^2 &= -4 \\ x - 1 &= \pm\sqrt{-4} \end{aligned}$$

$\boxed{\text{Thus there are no } x\text{-intercepts.}}$

y-intercepts: set $x = 0$:

$$\begin{aligned}(0-1)^2 + (y+4)^2 &= 12 \\ 1 + (y+4)^2 &= 12 \\ (y+4)^2 &= 11 \\ y+4 &= \pm\sqrt{11} \\ y &= -4 \pm \sqrt{11}\end{aligned}$$

Thus the y-intercepts are $-4 \pm \sqrt{11}$.

Graph: